Manifestations of acoustical activity in TeO_2 crystals: The appearance of circularly phase difference

Zapeka B., Mys O. and Vlokh R.

Vlokh Institute of Physical Optics, 23 Dragomanov Street, 79005 Lviv, Ukraine, vlokh@ifo.lviv.ua

Received: 09.03.2016

Abstract. We describe the effect of acoustical activity existing in TeO_2 crystals under the condition of equal propagation velocities of the quasi-transverse and quasi-longitudinal waves. Relations for the phase velocities and the ellipticities of eigenwaves are obtained. It is found that the eigenwaves excited by the quasi-longitudinal wave and one of the quasi-transverse waves become circularly polarized. The difference of the phase velocities for these waves caused by the acoustical activity leads to a circular phase difference and also to rotation of the displacement vector with respect to the displacement vector that excites the acoustic wave.

Keywords: acoustic waves, acoustical activity, polarization, ellipticity, TeO_2 crystals

PACS: 43.25.Ed, 78.20.Ek
UDC: 534-16+ 534.27+535.562

1. Introduction

Acoustical activity [1] represents an analogue of a well-known optical activity effect. Both of the effects (acoustic or optical) appear in non-centrosymmetric media and manifest themselves in rotation of polarization plane of the transverse acoustic or optical waves propagating through these media. Of course, the polarization plane rotation is only the simplest manifestation of the effects. When considering the acoustical or optical activities, we deal in fact with the effects of spatial dispersion so that the polarization plane rotation in either case depends upon the wave vector of the waves.

The acoustical activity has been predicted in the works [2–4] and experimentally detected in quartz crystals (see Ref. [1]). It has been found that the polarization plane rotation due to the acoustical activity can reach significant values (3–5 rad/cm or $(17.2–28.7)\times10^3$deg/m [1]) whenever the transverse acoustic wave with the frequency belonging to the region 1.05–1.4 GHz propagates along a three-fold symmetry axis of quartz. In other words, the magnitudes of the acoustical and optical activity effects are comparable. Indeed, the module of the optical rotation for the same quartz crystals is equal to 17.32×10^3deg/m at the light wavelength 656.3 nm [5].

As evidenced by the Brillouin scattering results [6], a linearly polarized phonon excited in quartz, with its wave vector parallel to the three-fold axis, splits into two circularly polarized acoustic eigenwaves that propagate with different velocities. The difference is about 1% at 30 GHz. Notice that the three-fold axis in the crystalline quartz is parallel to ‘acoustic axis’ (i.e., the direction along which the two linearly polarized transverse waves with the orthogonal polarizations have the same phase velocity). In other words, under propagation of the transverse acoustic waves along the acoustic axes, the acoustical activity effect manifests itself in the same manner as the optical activity. The result is splitting of the input linearly polarized wave into the two circularly polarized ones, which propagate with different velocities. Having reached the outgoing face of a crystal sample, these waves are superimposed and the initial linearly polarized wave is restored, although with the displacement vector rotated by some angle with respect to the input (exciting) wave.

The acoustical activity has been studied in a number of materials, e.g. $Bi_{12}GeO_{20}$ [7], tellurium [8] and some liquid crystals [9]. On the example of $LiNbO_3$, it has been found that the acoustical activity affects the efficiency of acoustooptic diffraction [10]. As shown earlier (see, e.g., Ref. [2]), the acoustical activity manifests itself in pure rotation of the acoustic polarization plane when the transverse acoustic wave propagates along the three-fold or higher-symmetry axes which, at the same time, represent the acoustic axis. This holds true for the non-centrosymmetric crystals involving no symmetry mirror planes or inversion axes. Then the point symmetry groups suitable for detecting the acoustical activity in its simplest form are as follows: 3, 4, 6, 32, 422, 622, 23, and 432.

As a matter of fact, the acoustical activity is not forbidden in the crystals belonging to some other acentric symmetry groups – or under propagation of the acoustic waves in any other directions. The maximal number of nonzero independent components available in the fourth-rank axial tensor with the internal symmetry $\varepsilon V[V]^3$, which describes the acoustical activity, is equal to 30 for all the acentric point symmetry groups [11]. The structure of the tensor has also been obtained in Refs. [12, 13]. However, the authors of those works have dealt with the tensor of the internal symmetry $\varepsilon V[V]^2 V$, which describes the ‘total’ acoustical activity effect, including a so-called ‘weak activity’. The tensor includes 45 nonzero independent components. Notice that the weak acoustical activity is analogous to the weak optical activity effect, being described by the antisymmetric axial second-rank tensor. It has not yet been detected experimentally [14]. Although the acoustic materials of all types support propagation of the longitudinal acoustical waves and so detecting of the weak acoustical activity seems to be easier than for the case of the weak optical activity, in the present work we will still focus on the acoustical activity tensor in its simplest form, i.e. the tensor containing 30 independent components. This approach can help clearly separate the acoustical activity effects of different natures.

Note that even simple comparison of the ranks of the acoustical and optical activity tensors suggests that the former effect has to be more complicated in its manifestations than its optical analogue. The simplest experimental manifestation, i.e. the rotation of polarization plane, has been considered in the works [11, 13] and the phenomenological relations describing the rotation angle have been obtained. In addition, the study [11] has demonstrated that the transverse and elliptically polarized longitudinal waves can interact in case of the acoustical activity. Nonetheless, no detailed description of the ellipticity of interacting eigenwaves has been reported up to now. Tellurium dioxide crystals represent one of suitable candidates for such a description, since the phase velocities of one of the quasi-transverse waves and the quasi-longitudinal wave can become equal for certain propagation directions (see Ref. [5]). The main goal of this work is to derive the ellipticity of acoustic eigenwaves in the presence of acoustical activity in TeO_2 crystals and to analyze the coupling of quasi-longitudinal and quasi-transverse acoustic waves in this material.

2. Results of analysis

Let us introduce a phenomenological description of the acoustical activity effect. After accounting for the first-order spatial dispersion, one can write the Hooke’s law as (see, e.g., Ref. [6])

$$\sigma_{ij} = C_{ijkl} e_{kl} + B_{ijklm} \frac{\partial e_{kl}}{\partial X_m} = C_{ijkl} \frac{\partial u_k}{\partial X_l} + B_{ijklm} \frac{\partial^2 u_k}{\partial X_l \partial X_m} , \tag{1}$$

where σ_{ij} and e_{kl} are respectively the stress and the strain caused by the acoustic wave, X_l and X_m the coordinates, C_{ijkl} is the elastic stiffness tensor, u_k the displacement vector, and B_{ijklm} the polar fifth-rank tensor describing the acoustical gyration. Then the elastodynamical equation taking the spatial dispersion into account may be written as

$$C_{ijkl}\frac{\partial^2 u_k}{\partial X_i \partial X_l} + B_{ijklm}\frac{\partial^3 u_k}{\partial X_i \partial X_l \partial X_m} = \rho\frac{\partial^2 u_j}{\partial t^2}, \tag{2}$$

where ρ is the material density and t the time coordinate. For the simplest case of plane waves with the unit polarization vector p_k, the amplitude *A*, the wave vector *m*, the velocity *v* and the wavelength Λ ($u_k = Ap_k e^{\frac{2\pi}{\Lambda}i(mr\text{-}vt)}$), Eq.(2) reads as

$$\left(C_{ijkl}m_i m_l + \frac{2\pi}{\Lambda}iB_{ijklm}m_i m_l m_m\right)p_k = \rho v^2 p_j, \tag{3}$$

where $M_{ik}^a = C_{ijkl}m_i m_l + \frac{2\pi}{\Lambda}iB_{ijklm}m_i m_l m_m$ is the Christoffel tensor that accounts for the acoustical activity. The elastic stiffness tensor is Hermitian and includes antisymmetric imaginary part, $\frac{2\pi}{\Lambda}iB_{ijklm}m_i m_l m_m$. Hence, the relation $B_{ijklm} = -B_{klijm}$ holds true and the tensor reveals the internal symmetry $\{[V^2]^2\}V$. However, the tensor is symmetric with respect to permutations of the indices *i*, *l* and *m* that correspond to the wave vectors. Then the internal symmetry of B_{ijklm} reduces to $\{V^2\}[V^3]$. As a consequence, it can be rewritten in terms of the axial fourth-rank acoustical gyration tensor g_{silm} with the internal symmetry $\varepsilon V[V^3]$:

$$g_{silm} = \frac{\pi}{\Lambda}\delta_{sjk}B_{ijklm}, \quad \frac{2\pi}{\Lambda}B_{ijklm} = \delta_{jks}g_{silm}, \tag{4}$$

where δ_{sjk} is the unitary, fully antisymmetric axial Levi–Civita tensor. The axial vector of the acoustical activity is determined by the acoustical gyration tensor as

$$\phi_s = g_{silm}m_i m_l m_m. \tag{5}$$

Finally, one can reduce Eq. (3) to the following form:

$$\left(M_{jk} + i\delta_{jks}\phi_s\right)p_k = \rho v^2 p_j, \tag{6}$$

where M_{jk} is the Christoffel tensor that does not take the acoustical activity into account.

Let us consider Eq. (6) in a more detail. In general, it can be presented as a system of equations (see, e.g., Ref. [14]):

$$\begin{cases} \rho v^2 p_1 = M_{11}p_1 + i\phi_3 p_2 - i\phi_2 p_3 \\ \rho v^2 p_2 = -i\phi_3 p_1 + M_{22}p_2 + i\phi_1 p_3 \\ \rho v^2 p_2 = i\phi_2 p_1 - i\phi_1 p_2 + M_{33}p_3 \end{cases}, \tag{7}$$

where v_{0l} (l = 1, 2, 3) are the acoustic wave velocities which obey the relations $\rho v_{01}^2 p_1 = M_{11}p_1$, $\rho v_{02}^2 p_2 = M_{22}p_2$ and $\rho v_{03}^2 p_3 = M_{33}p_3$ under the condition of absence of the acoustical activity effect, and v implies the acoustic wave velocities that account for the latter effect. As a result, we present the system of Eqs. (7) as

$$\begin{cases} \rho(v_{01}^2 - v^2)p_1 + i\phi_3 p_2 - i\phi_2 p_3 = 0 \\ -i\phi_3 p_1 + \rho(v_{02}^2 - v^2)p_2 + i\phi_1 p_3 = 0 \\ i\phi_2 p_1 - i\phi_1 p_2 + \rho(v_{03}^2 - v^2)p_3 = 0 \end{cases}. \tag{8}$$

In fact, the solutions of this system determine the velocities of the acoustic eigenwaves and their ellipticities.

As already mentioned, the simplest case of the acoustical activity has already been described in the literature (see, e.g., Ref. [11]). It happens when the transverse acoustic wave propagates along the three-fold (or higher-order) symmetry axis which, at the same time, represents the acoustic axis. For example, when the transverse acoustic wave with the wave vector component m_3 is excited in the acentric crystals belonging to the middle symmetry systems, Eq. (5) may be written as $\phi_3 = g_{3333}m_3^3$. According to Eq. (6), the change in the polarization of this wave should obey the relations $p_1 \sim [\phi_3 \times p_2]$ or $p_2 \sim [\phi_3 \times p_1]$. Thus, the system of Eqs. (8) may be represented as

$$\begin{cases} \rho(v_{01}^2 - v^2)p_1 + i\phi_3 p_2 = 0 \\ -i\phi_3 p_1 + \rho(v_{02}^2 - v^2)p_2 = 0 \\ \rho(v_{03}^2 - v^2)p_3 = 0 \end{cases}. \tag{9}$$

If the wave vector component m_3 is parallel to the higher-symmetry axis, the eigenwave with the polarization p_3 (i.e., the longitudinal wave) suffers no change in its propagation velocity due to the acoustical activity, i.e. $v_3 = v_{03}$. In this case the system of Eqs. (9) is simplified to

$$\begin{cases} \rho(v_{01}^2 - v^2)p_1 + i\phi_3 p_2 = 0 \\ -i\phi_3 p_1 + \rho(v_{02}^2 - v^2)p_2 = 0 \end{cases}. \tag{10}$$

The two solutions that correspond to the velocities of the transverse waves are

$$\begin{aligned} v_{1,2}^2 &= \frac{1}{2}\left[(v_{01}^2 + v_{02}^2) \pm \sqrt{(v_{01}^2 + v_{02}^2)^2 - 4v_{01}^2 v_{02}^2 + 4\phi_3^2 / \rho^2}\right]\Bigg|_{v_{01}=v_{02}=v_0} \\ &= v_0^2 \pm \frac{\phi_3}{\rho}. \end{aligned} \tag{11}$$

The ellipticities κ of these waves are defined by the relation $-i\kappa = p_1 / p_2$, being equal to ± 1. This describes the two circularly polarized waves with the opposite rotation directions of the displacement vector. The waves that pass through the sample with the thickness d acquire the phase difference $\Delta_c = \phi_3 \omega d / \rho v_0^3$. Hence, the angle of rotation of the polarization plane is equal to $\varphi = \frac{\Delta}{2} = \frac{\phi_3 \omega}{2\rho v_0^3} d$.

The situation is more complex in the crystals that belong, e.g., to the symmetry groups 222, mm2, 2 and m ($2 \parallel X_3$, $\mathrm{m} \perp X_3$). Here the velocities $v_{01} \neq v_{02}$ and Eq. (11) results in

$$v_{1,2}^2 = \frac{1}{2}\left[(v_{01}^2 + v_{02}^2) \pm (v_{01}^2 - v_{02}^2)\sqrt{1 + \frac{4\phi_3^2}{\rho^2 (v_{01}^2 - v_{02}^2)^2}} \right]\Bigg|_{v_{01} \neq v_{02}}$$

$$\simeq \frac{1}{2}\left[(v_{01}^2 + v_{02}^2) \pm \left((v_{01}^2 - v_{02}^2) + \frac{2\phi_3^2}{\rho^2 (v_{01}^2 - v_{02}^2)} \right) \right] \quad (12)$$

$$= \begin{cases} v_{01}^2 + \dfrac{\phi_3^2}{\rho^2 (v_{01}^2 - v_{02}^2)} \\ v_{02}^2 - \dfrac{\phi_3^2}{\rho^2 (v_{01}^2 - v_{02}^2)} \end{cases},$$

where $\frac{4\phi_3^2}{\rho^2 (v_{01}^2 - v_{02}^2)^2} \ll 1$. The ellipticity of these waves is given by the relation $-i\kappa = \frac{p_1}{p_2}$ (or $i\kappa = \frac{p_1}{p_2}$), i.e.

$$\kappa = \begin{cases} \rho(v_{01}^2 - v_{02}^2)/\phi_3 \\ \phi_3 / \rho(v_{01}^2 - v_{02}^2) \end{cases}. \quad (13)$$

In fact, the polarizations ellipses of the waves are characterized by high eccentricity.

The analogous behaviours of the optical and acoustic waves in the presence of the gyration effect suggest that there is no reason for their different analytical description. As a result, when the acoustic wave is excited with the displacement vector parallel to the eigenvectors of the Christoffel tensor, the polarization ellipse of the emergent wave should oscillate with the angular amplitude χ defined by the known relation

$$\tan 2\chi = -\frac{\frac{2\kappa}{1+\kappa^2}\sin\Delta_e}{\left(\frac{1-\kappa^2}{1+\kappa^2}\right)^2 + \left(\frac{2\kappa}{1+\kappa^2}\right)^2 \cos\Delta_e}, \quad (14)$$

where Δ_e denotes the elliptical phase difference. The ellipticity ($b/a = \tan\gamma$) of the emergent wave is given by

$$\tan\gamma = -2\kappa \frac{1-\kappa^2}{(1-\kappa^2)^2}(1 - \cos\Delta_e). \quad (15)$$

Notice that the elliptical phase difference satisfies the relation

$$(\Delta_e)^2 = (\Delta_l)^2 + (\Delta_c)^2, \quad (16)$$

where $\Delta_l = \omega d(v_{01} - v_{02})/v_{01}v_{02}$ implies the phase difference between linearly polarized waves in the absence of acoustical activity. In fact, Eq. (16) represents a so-called superposition principle, which is well known from the crystal optics [15].

However, similarities in the description of the gyrotropy in crystal acoustics and crystal optics are limited only to the two cases of coupling of the transverse waves due to the acoustical activity, which are described above. The third, longitudinal wave is not coupled with these transverse waves in the example analyzed above. Notice also that the system of Eqs. (8) can be solved analytically in general case, although the relevant solutions are cumbersome and cannot be simplified for their further analysis. On the other hand, the coupling of the longitudinal and transverse waves in acoustically active media can be demonstrated in the best way for the crystals in which the velocity of one of the quasi-transverse waves is equal to the velocity of the quasi-longitudinal wave. Though the velocities of the longitudinal waves are usually higher than those of the transverse waves, this unique situation is really observed in TeO_2 [5]. When the acoustic waves propagate in the principal planes X_1X_3 or X_2X_3 under the angle Θ = 80 deg with respect to the X_3 axis, the velocity of the quasi-transverse acoustic wave polarized perpendicularly to these planes is equal to the velocity of the quasi-longitudinal wave (3014 m/s – see, e.g., Ref. [5]). The cross sections of the acoustic-wave velocity surfaces by the principal planes X_1X_3 and X_2X_3 for TeO_2 are presented at Fig. 1. These have been constructed basing on the Christoffel equation that neglects the acoustical activity, using the elastic stiffness coefficients reported in Ref. [16] (C_{11} = 5.32, C_{12} = 4.86, C_{13} = 2.12, C_{33} = 10.85, C_{44} = 2.44 and $C_{66} = 5.52\times10^{10}$ N/m^2).

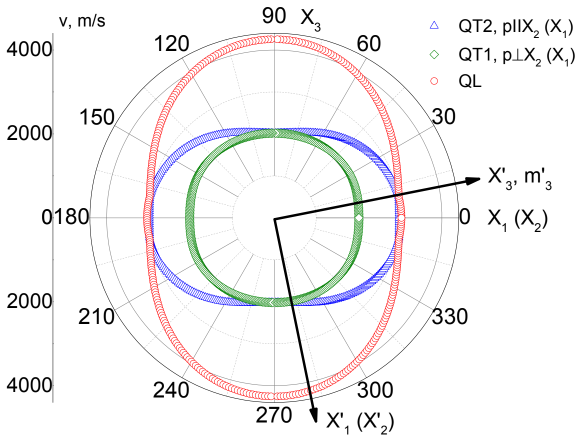

Fig. 1. Cross sections of acoustic-wave velocity surfaces by the principal planes X_1X_3 and X_2X_3 for TeO_2 crystals: QL corresponds to quasi-longitudinal wave, and QT1 and QT2 to quasi-transverse waves.

To proceed, we have to transform the tensor g_{silm}, which has been written in Ref. [14] in the crystallographic coordinate system $X_1X_2X_3$, to the system rotated around the X_2 axis by the angle Θ in such a manner that the axis X'_3 becomes parallel to the acoustic wave vector and to the direction where the velocities of the quasi-transverse and quasi-longitudinal waves are equal (see Fig. 1). Then the gyration tensor components in the coordinate system $X_1X_2X_3$ given by

	$m_3m_3m_3$
ϕ_1	0
ϕ_2	0
ϕ_3	g_{3333}

should be rewritten in the $X'_1X_2X'_3$ system as

	$m'_3m'_3m'_3$
ϕ'_1	g'_{1333}
ϕ'_2	0
ϕ'_2	g'_{3333}

, (17)

where

$$g'_{1333} = (g_{1111} - g_{3311})\cos\Theta\sin^3\Theta + (g_{1133} - g_{3333})\sin\Theta\cos^3\Theta$$
$$= 0.1659\,(g_{1111} - g_{3311}) + 0.0052(\,g_{1133} - g_{3333}), \quad (18)$$

and

$$g'_{3333} = g_{1111}\sin^4\Theta + g_{3333}\cos^4\Theta + (g_{3311} + g_{1133})\sin^2\Theta\cos^2\Theta \\ = 0.9406\, g_{1111} + 0.0009\, g_{3333} + 0.0292\, (g_{3311} + g_{1133}). \tag{19}$$

Accordingly, the system of Eqs. (8) becomes as follows:

$$\begin{cases} \rho(v_{01}^2 - v^2)p_1 + i\phi_3 p_2 = 0 \\ -i\phi_3 p_1 + \rho(v_0^2 - v^2)p_2 + i\phi_1 p_3 = 0 \\ -i\phi_1 p_2 + \rho(v_0^2 - v^2)p_3 = 0 \end{cases}, \tag{20}$$

with $v_{02} = v_{03} = v_0 = 3014$ m/s , $v_{01} = 2030$ m/s and $\rho = 5990$ kg/m^3 [16].

For further numerical simulations we are to fix the parameters ϕ_1 and ϕ_3. The relevant data for TeO_2 is not available in the literature and so we accept ϕ_1 and ϕ_3 to be of the same order of magnitude as those typical for the quartz crystals [14] (10^7 kg/ms^2 and 5×10^7 kg/ms^2, respectively). We have $\frac{cbd}{6} - \frac{c^2b^2}{108} - \frac{c^3}{27} + \frac{b^3d}{27} + \frac{d^2}{4} < 0$ for the discriminant associated with the system of Eqs. (20), where $b = v_{01}^2 + 2v_0^2$, $c = \frac{\phi_1^2}{\rho^2} + \frac{\phi_3^2}{\rho^2} - 2v_{01}^2v_0^2 - v_0^4$ and $d = v_0^4v_{01}^2 - \frac{\phi_1^2v_{01}^2 + \phi_3^2v_0^2}{\rho^2}$. Then there are three real solutions of Eqs. (20):

$$v^2_{QL} = \frac{b}{3} + \frac{\frac{b^2}{9} + \frac{c}{3}}{\left(\frac{d}{2} + \frac{b^3}{27} + \frac{cb}{6} + \sqrt{\frac{cbd}{6} - \frac{c^2b^2}{108} - \frac{c^3}{27} + \frac{b^3d}{27} + \frac{d^2}{4}}\right)^{1/3}} + \left(\frac{d}{2} + \frac{b^3}{27} + \frac{cb}{6} + \sqrt{\frac{cbd}{6} - \frac{c^2b^2}{108} - \frac{c^3}{27} + \frac{b^3d}{27} + \frac{d^2}{4}}\right)^{1/3}, \tag{21}$$

$$\begin{aligned} v^2_{QT1} = \frac{b}{3} &- \frac{\frac{b^2}{9} + \frac{c}{3}}{2\left(\frac{d}{2} + \frac{b^3}{27} + \frac{cb}{6} + \sqrt{\frac{cbd}{6} - \frac{c^2b^2}{108} - \frac{c^3}{27} + \frac{b^3d}{27} + \frac{d^2}{4}}\right)^{1/3}} \\ &- \frac{1}{2}\left(\frac{d}{2} + \frac{b^3}{27} + \frac{cb}{6} + \sqrt{\frac{cbd}{6} - \frac{c^2b^2}{108} - \frac{c^3}{27} + \frac{b^3d}{27} + \frac{d^2}{4}}\right)^{1/3} \\ &- \frac{\sqrt{3}}{2}i\left[\frac{\frac{b^2}{9} + \frac{c}{3}}{\left(\frac{d}{2} + \frac{b^3}{27} + \frac{cb}{6} + \sqrt{\frac{cbd}{6} - \frac{c^2b^2}{108} - \frac{c^3}{27} + \frac{b^3d}{27} + \frac{d^2}{4}}\right)^{1/3}} - \left(\frac{d}{2} + \frac{b^3}{27} + \frac{cb}{6} + \sqrt{\frac{cbd}{6} - \frac{c^2b^2}{108} - \frac{c^3}{27} + \frac{b^3d}{27} + \frac{d^2}{4}}\right)^{1/3}\right], \end{aligned} \tag{22}$$

$$v^2_{QT2}=\frac{b}{3}-\frac{\frac{b^2}{9}+\frac{c}{3}}{2\left(\frac{d}{2}+\frac{b^3}{27}+\frac{cb}{6}+\sqrt{\frac{cbd}{6}-\frac{c^2b^2}{108}-\frac{c^3}{27}+\frac{b^3d}{27}+\frac{d^2}{4}}\right)^{1/3}}$$
$$-\frac{1}{2}\left(\frac{d}{2}+\frac{b^3}{27}+\frac{cb}{6}+\sqrt{\frac{cbd}{6}-\frac{c^2b^2}{108}-\frac{c^3}{27}+\frac{b^3d}{27}+\frac{d^2}{4}}\right)^{1/3} \tag{23}$$
$$+\frac{\sqrt{3}}{2}i\left[\frac{\frac{b^2}{9}+\frac{c}{3}}{\left(\frac{d}{2}+\frac{b^3}{27}+\frac{cb}{6}+\sqrt{\frac{cbd}{6}-\frac{c^2b^2}{108}-\frac{c^3}{27}+\frac{b^3d}{27}+\frac{d^2}{4}}\right)^{1/3}}-\left(\frac{d}{2}+\frac{b^3}{27}+\frac{cb}{6}+\sqrt{\frac{cbd}{6}-\frac{c^2b^2}{108}-\frac{c^3}{27}+\frac{b^3d}{27}+\frac{d^2}{4}}\right)^{1/3}\right],$$

Eqs. (21)–(23) are rather cumbersome and cannot be easily simplified. The velocities of the acoustic waves calculated from these relations are as follows: $v_{QL}=3014.30$ m/s, $v_{QT1}=2030.02$ m/s and $v_{QT2}=3013.80$ m/s. Therefore the acoustical activity changes little the velocities of the acoustic waves.

It would be instructive to verify if the same result can be obtained after 'stepwise' transformation of the Christoffel tensor, from which the system of Eqs. (20) follows, into the eigen coordinate system. This transformation is realized by (i) eliminating the component $M_{23}=i\phi_1$ and then (ii) eliminating the $M_{12}=i\phi_3$ component. The final relations obtained for the acoustic wave velocities acquire the following form:

$$v^2_{QL}=\frac{1}{2\rho}\left(\rho(v_0^2+v_{01}^2)+\phi_1+\sqrt{\rho^2(v_0^2-v_{01}^2)^2+2\phi_1\rho(v_0^2-v_{01}^2)+\phi_1^2+4\phi_3^2}\right)$$
$$\simeq v_0^2+\frac{\phi_1}{2\rho}+\frac{\phi_1\rho(v_0^2-v_{01}^2)+\phi_1^2+4\phi_3^2}{2\rho^2(v_0^2-v_{01}^2)}, \tag{24}$$

$$v^2_{QT1}=\frac{1}{2\rho}\left(\rho(v_0^2+v_{01}^2)+\phi_1-\sqrt{\rho^2(v_0^2-v_{01}^2)^2+2\phi_1\rho(v_0^2-v_{01}^2)+\phi_1^2+4\phi_3^2}\right)$$
$$\simeq v_{01}^2+\frac{\phi_1}{2\rho}-\frac{\phi_1\rho(v_0^2-v_{01}^2)+\phi_1^2+4\phi_3^2}{2\rho^2(v_0^2-v_{01}^2)}, \tag{25}$$

$$v^2_{QT2}=v_0^2-\frac{\phi_1}{\rho}. \tag{26}$$

The acoustic wave velocities calculated using Eqs. (24)–(26) are essentially the same as those derived with Eqs. (21)–(23): $v_{QL}=3014.30$ m/s, $v_{QT1}=2030.02$ m/s and $v_{QT2}=3013.80$ m/s. Hence, one can readily use Eqs. (24)–(26) while describing the changes occurring in the acoustic wave velocities due to the acoustical activity.

Now we consider the changes that occur in the polarization of the acoustic eigenwaves due to the acoustical activity for the geometry of wave propagation in TeO_2 considered above. It follows from the system of Eqs. (20) that the ellipticity of the eigenwaves is given by

$$\frac{p_2}{p_1} = i\kappa_{QT1} = -\frac{\phi_1^2 + 4\phi_3^2}{i2\phi_3\rho(v_0^2 - v_{01}^2)}$$
$$\frac{p_2}{p_3} = i\kappa_{QT2} = \frac{1}{i} \tag{27}$$
$$\frac{p_2}{p_3} = i\kappa_{QL} = -\frac{\rho}{i\phi_1}\frac{2\phi_1\rho(v_0^2 - v_{01}^2) + \phi_1^2 + 4\phi_3^2}{2\rho^2(v_0^2 - v_{01}^2)}\Bigg|_{\phi_1^2,\, 4\phi_3^2 \ll \phi_1\rho(v_0^2 - v_{01}^2)} \simeq \frac{-1}{i}.$$

In other words, if the acoustic wave QT1 is excited, the polarization ellipse of the wave propagating in the sample lies in the $X_1^{'}X_2$ plane and the corresponding ellipticity reads as

$$\kappa_{QT1}^{X_1^{'}X_2} = \frac{\phi_1^2 + 4\phi_3^2}{2\phi_3\rho(v_0^2 - v_{01}^2)}. \tag{28}$$

This ellipticity is quite small and depends on the difference of velocities of the two quasi-transverse acoustic waves. Moreover, the polarization ellipse is elongated along the $X_1^{'}$ axis (see Fig. 2a). Notice that the wave propagates with no splitting. When the other wave QT2 is excited, circularly polarized waves can appear, with the polarization circle belonging to the $X_3^{'}X_2$ plane (see Fig. 2b). The corresponding ellipticity is determined as

$$\kappa_{QT2}^{X_3^{'}X_2} = -1. \tag{29}$$

Notice also that the polarization circle belonging to the $X_3^{'}X_2$ plane is left-handed.

Now we consider the effect of the acoustical activity on the polarization of the quasi-longitudinal wave QL. When the quasi-longitudinal wave is excited, the eigenwave is almost circularly polarized, with the ellipticity

$$\kappa_{QL}^{X_3^{'}X_2} \simeq 1. \tag{30}$$

Then the polarization circle belongs to the $X_3^{'}X_2$ plane and the circular polarization has the opposite sign when compared to the polarization occurring in the case of the QT2 wave (see Fig. 2c).

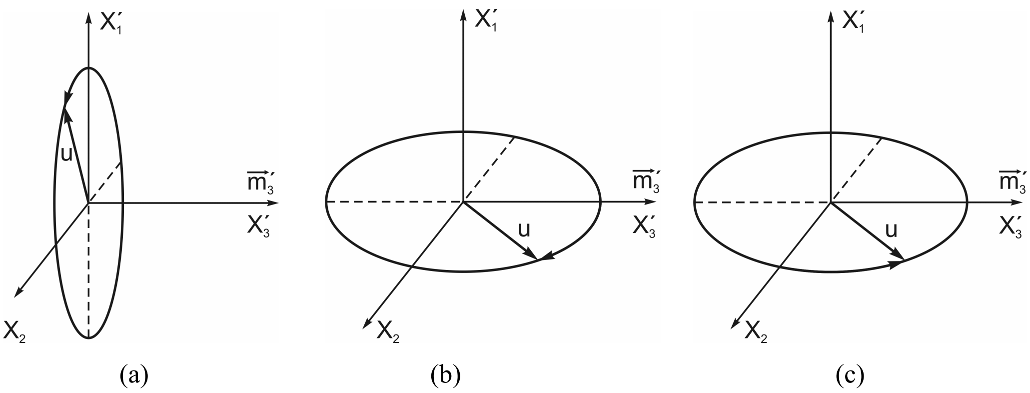

Fig. 2. Schematic view of polarization ellipses for the acoustic waves QT1 (a), QT2 (b) and QL (c) that propagate in TeO_2 crystals.

When the acoustic wave with nonzero projection of the displacement vector on the $X_3^{'}$ and X_2 axes is excited in the sample, it splits into two circularly polarized waves. These waves propagate with close enough velocities (the difference $\Delta v = v_{QL} - v_{QT2} = 0.5$ m/s is very small) and acquire the circular phase difference $\Delta_c = \omega d \Delta v / v_{QL} v_{QT2}$ after reaching the output face of the sample. At the frequency 10 GHz and the sample thickness as small as 10^{-2} m, this phase difference is equal to 5.5 rad, while the angle of rotation of the displacement vector in the $X_3^{'} X_2$ plane with respect to the displacement vector of the incoming wave is two times smaller (2.25 rad or 128.98 deg). Therefore, if one excites in the sample the acoustic wave of which displacement vector has two nonzero components u_{X_2} and $u_{X_3^{'}}$, the ratio of these components for the outgoing wave would change (see Fig. 3). This can be implemented experimentally with a material in which the quasi-transverse (or quasi-longitudinal) wave is characterized by high enough deviation angle from the purely transverse (or longitudinal) polarization. As a result, using the effect of acoustical activity and choosing appropriately the thickness of TeO_2 sample, one can transform quasi-longitudinal (quasi-transverse) acoustic waves into purely longitudinal (purely transverse) waves, or vice versa.

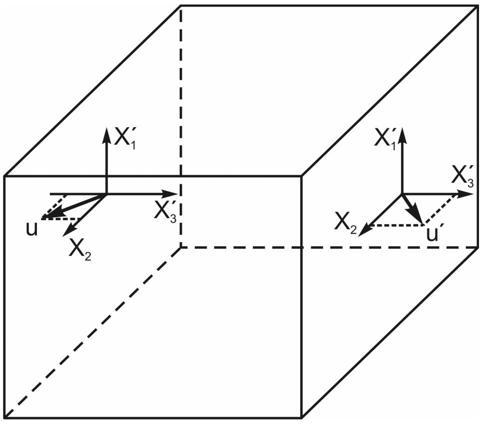

Fig. 3. Schematic representation of rotation of the displacement vector due to the acoustical activity effect.

3. Conclusions

In the present work we have described the effect of acoustical activity in TeO_2 crystals, which occurs in case when the propagation velocities of the quasi-transverse and quasi-longitudinal acoustic waves become equal. The relations for the phase velocities and the ellipticities of the eigenwaves have been obtained. We have found that, due to the acoustical activity, the quasi-transverse and quasi-longitudinal waves are coupled with each other. When propagating in a crystalline medium, they produce circularly polarized eigenwaves. It is shown that the phase velocity difference for these waves, which is caused by the acoustical activity, leads to the appearance of circular phase difference and so to rotation of the displacement vector with respect to the input acoustic wave. We have also demonstrated that the principle of superposition of the linear and circular birefringences, which is well-known in the crystal optics, remains valid for the crystal acoustics.

References

1. Pine A S, 1970. Direct observation of acoustical activity in α-quartz. Phys. Rev. B. **2**: 2049–2054.

2. Silin V P, 1960. Contribution to the theory of absorption of ultrasound in metals. JETP. **11**: 703–707.
3. Andronov A A, 1960. On the natural rotation of the polarization plane of sound. Izv. Vyshikh Uchebn. Zavedeniy, Ser. Radiofiz. **3**: 645–649.
4. Portigal D L and Burstein E, 1968. Acoustical activity and other first-order spatial dispersion effects in crystals. Phys. Rev. **170**: 674–678.
5. Shaskolskaya M P. Acoustic crystals. Moscow: Nauka (1982).
6. Pine A S, 1971. Linear wave-vector dispersion of the shear-wave phase velocity in quartz. J. Acoust. Soc. Amer. **49**: 1026–1029.
7. Lin C and Fang Tao, 1985. A study of acoustical activity of $Bi_{12}GeO_{20}$. Solid State Commun. **54**: 803–806.
8. Shen Zhigong, Zhao Jinkui, Lin Quan (C. Lin), Jialing Yu and Changgui Yi, 1989. Acoustical activity in tellurium. Solid State Commun. **72**: 1027–1031.
9. Moritz Elan, 1978. Acoustical activity in liquid crystals. Mol. Cryst. Liq. Cryst. **49**: 7–11.
10. Akhmedzhanov Farkhad R, 2015. Acoustical activity of lithium niobate crystals. J. Acoust. Soc. Amer. **138**: 1940.
11. Vuzhva A D and Lyamov V E, 1977. Acoustical activity and other effects, caused by the spatial dispersion in crystals. Kristallografiya. **22**: 131–137.
12. Kumaraswamy K and Krishnamurthy N, 1980. The acoustic gyrotropic tensor in crystals. Acta Cryst. A. **36**: 760–762.
13. Bhagwat K V, Wadhawan V K and Subramanian R, 1986. A new fourth-rank tensor for describing the acoustical activity of crystals. J. Phys. C: Solid State Phys. **19**: 345–357.
14. Sirotin Yu I and Shaskolskaya M P. Fundamentals of crystal physics. Moscow: Nauka (1979).
15. Konstantinova A F, Grechushnikov B N, Bokut B V and Velyashko E G. Optical properties of crystals. Minsk: Nauka i Technika (1995).
16. Uchida N and Ohmachi Y, 1969. Elastic and photoelastic properties of TeO_2 single crystal. J. Appl. Phys. **40**: 4692–4695.

Zapeka B., Mys O. and Vlokh R. 2016. Manifestations of acoustical activity in TeO_2 crystals: The appearance of circularly phase difference. Ukr.J.Phys.Opt. **17**: 47 – 57.

Анотація. *Описано явище акустичної активності в кристалах TeO_2 за умови однакових швидкостей поширення квазі-поперечної і квазі-поздовжньої хвиль. Отримано співвідношення для швидкостей і еліптичностей власних хвиль. Установлено, що власні хвилі, які виникають внаслідок збудження квазі-поздовжньої і однієї з квазі-поперечних хвиль, є циркулярно поляризованими. Різниця швидкостей цих хвиль, спричинена акустичною активністю, приводить до появи циркулярної різниці фаз і повороту вектора зміщення по відношенню до вектора зміщення, що збуджує акустичну хвилю.*