
Moving domain walls with Néel defects in optical oscillator

Yaparov V. V. and Taranenko V. B.

International Center “Institute of Applied Optics” of the National Academy of Sciences of Ukraine, 10-G Kudryavska Street, 04053 Kyiv, Ukraine,
yaparovwork@mail.ru

Received: 10.07.2015

Abstract. We show both numerically and experimentally that the optical oscillator with combined parametric and laser gains supports formation of translationally or/and rotationally moving composite topological structures appearing in the form of optical domain walls, with the point defects similar to Néel topological defects available in ferromagnetics and liquid crystals.

Keywords: pattern formation, optical vortices, optical domain walls

PACS: 47.54.-r, 42.65.Sf, 42.65.Hw

UDC: 535.2/8

1. Introduction

Optical oscillators with laser or parametric gains can generate optical fields with nontrivial topological defects of phase such as screw or edge phase dislocations [1]. The screw dislocations, or vortices, which represent point topological defects of zero intensity around which the phase changes by 2π , occur as particular solutions of the laser equations [2]. They have been observed experimentally in the multimode lasers and laser-like non-degenerate wave-mixing oscillators [3–5]. The formation of edge phase dislocations, or optical domain walls, has been predicted for the degenerate parametric oscillators [6, 7]. The walls that exist in the form of dark lines separating two spatial domains of equal light intensity and opposite phases have been experimentally demonstrated for a phase-bistable four-wave-mixing oscillator [8]. Two types of the optical domain walls have been found: (i) static Ising domain walls for which the phase jumps abruptly by π and (ii) moving Bloch domain walls with a smooth change in the phase. A transition between those walls can occur with detuning of the oscillator cavity [9] and the optical vortices can be converted into the optical domain wall structures via parametric rocking [10]. This has been implemented experimentally in the two-wave-mixing oscillator, using injection of an amplitude-modulated laser beam into a cavity [11].

In this work we investigate a spontaneous pattern formation in an active cavity with mixed laser and parametric gains, and demonstrate for the first time the existence, in such a cavity, of a new type of optical topological structures, which is a translationally or/and rotationally moving two-dimensional optical domain wall with Néel point defects (NDs).

2. Modelling and analysis

We consider an optical oscillator containing a laser gain g and a parametric gain γ , both being located between plane parallel mirrors. An optical field is formed inside this oscillator, starting from spontaneous emission when the gain exceeds a threshold. Since the field changes weakly during propagation between the cavity mirrors, we use the mean-field approximation implying that

the optical field is averaged over the longitudinal coordinate z . The dynamics of the two-dimensional optical field (or the pattern) inside such an active cavity can be well described by the complex Ginzburg–Landau equation, which represents a universal description for parametrically excited waves of different natures. In order to take into account a saturable character of both the laser and parametric gains, we modify the complex Ginzburg–Landau equation to the following form:

$$\frac{\partial E}{\partial t} = \frac{g E + \gamma E^*}{1 + |E|^2} - \eta E + (d_{\text{Re}} + i d_{\text{Im}}) \nabla_{\perp}^2 E. \quad (1)$$

Here $E(x, y, t)$ denotes the optical field envelope, t the evolution variable (the time normalized by the photon life-time in the cavity), E^* the complex conjugate of E , η the coefficient of linear losses, d_{Re} the effective diffusion coefficient, d_{Im} the diffraction coefficient and $\nabla_{\perp}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ the transverse Laplace operator, whereas x and y are the dimensionless transverse coordinates normalized by the width of the first Fresnel zone of the cavity.

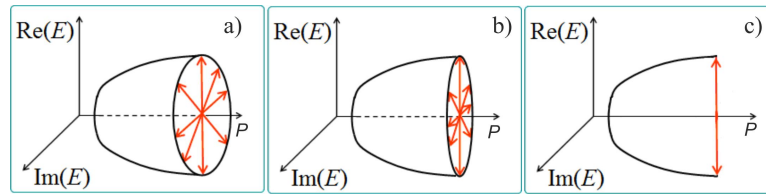


Fig. 1. Generation of field versus pump: (a) a complex isotropic field for the case of $\gamma = 0$, (b) a complex anisotropic field for $g, \gamma > 0$, and (c) a real field for $g = 0$.

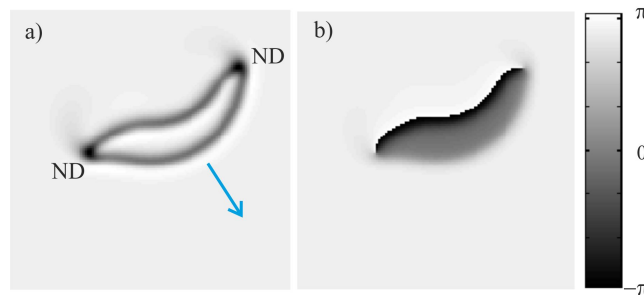


Fig. 2. Transverse spatial distributions of intensity (a) and phase (b) for the moving domain-wall loop with two NDs. The arrow indicates direction of translational motion. The parameters are as follows: $g = 2.12$, $\gamma = 0.237$ and $\eta = 1$.

First we analyze the homogeneous and steady-state solutions of Eq. (1). When $\gamma = 0$, Eq. (1) becomes phase invariant and so describes a dynamics of the class A laser. In this case the complex optical field vectors lean on the circle (i.e., the field is complex and isotropic), provided that the pump P is kept above a threshold (see Fig.1a). The phase invariance is broken when we have $g = 0$ because of a presence of the parametric term γE^* . Then Eq. (1) becomes phase bistable (see Fig. 1c). It describes a degenerate parametric oscillator for which the optical field is real. The homogeneous solutions have the two equivalent values, E_0 and $-E_0$, i.e. the phase can acquire only the two values, 0 and π , thus implying that the field is real. In the general case of $g, \gamma > 0$, the intracavity optical field becomes complex and anisotropic, as illustrated in Fig. 1b. This means that the phase of the field has two preferable values that differ by π from each other.

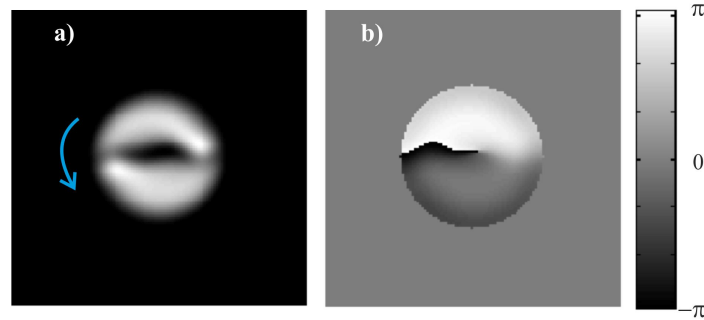


Fig. 3. Transverse spatial distributions of intensity (a) and phase (b) for the rotating domain wall with the ND. The arrow indicates direction of rotation. The parameters are as follows: $g = 1.2$, $\gamma = 0.063$ and $\eta = 1$.

To find inhomogeneous spatially extended structures of the optical field (the patterns) for different g/γ ratios, we solve Eq. (1) numerically using a split-step fast Fourier transform method with the periodic boundary conditions. For the finite Fresnel numbers of the cavity we introduce an aperture bounding the field in Eq. (1). As expected, the solutions of Eq. (1) for large (> 100) Fresnel numbers of the cavity acquire the form of optical vortices for the complex isotropic field ($\gamma = 0$) and the form of optical Ising domain walls for the real field ($g = 0$). However, the solution of Eq. (1) for the complex anisotropic field ($g, \gamma > 0$) gives a new optical composite topological structure in the shape of a loop (see Fig. 2). It consists of two Bloch domain walls connected with the two NDs [12]. The loop moves translationally (see Fig. 2) as a one-dimensional Bloch domain wall moves along its own transverse phase gradient [9]. The point is that the domain walls in the loop are oriented almost parallel to each other and their transverse phase gradients have the same signs. It is worth noting that all the structures formed in such a wide-aperture oscillator are independent of the boundary conditions.

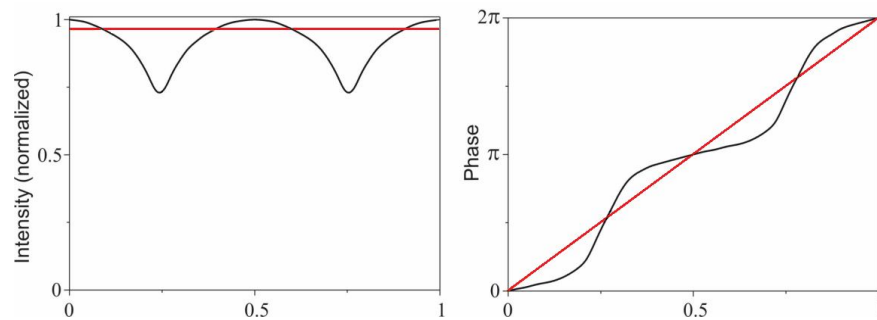


Fig. 4. Intensity (left panel) and phase (right panel) that occur when tracing around the centre of the structure with the ND (black lines) and in the case of optical vortex (red lines).

For moderate Fresnel numbers (~ 10) of the cavity, the boundary conditions play an important role. In this case the solution of Eq. (1) represents a composite rotating structure which is the domain wall leaning on the cavity aperture, with the ND located in the centre (see Fig. 3). To get a more detailed insight into this structure (see Fig. 3), we trace the changes in the intensity and the phase occurring in the circle line located around the central point of the structure (see black lines in Fig. 4). For comparison, the red lines represent the same dependences for the optical vortex.

One can see that there is certain similarity between these structures. However, there is also a difference in that the domain wall with the ND has two dips in the intensity over which the phase changes smoothly by π , as it takes place for the Bloch domain walls. The phase gradients of the

left and right parts of the domain wall are opposite to each other and the ND separates them (see Fig. 3), so that the phase has a shape of propeller blades in the three-dimensional representation. The phase histogram displayed in Fig. 5 confirms that we indeed deal with the Bloch-type domain walls.

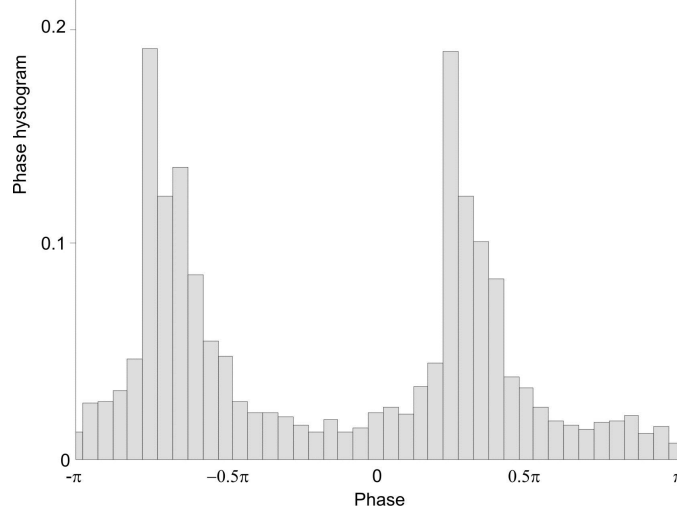


Fig. 5. Phase histogram corresponding to the structure shown in Fig. 3.

3. Experimental results and discussion

To demonstrate experimentally the existence of the optical domain walls with the topological NDs, we use a photorefractive ring oscillator (see Fig. 6). Two counter-propagating pump beams from a single-frequency laser (532 nm) illuminate a photorefractive crystal of strontium-barium niobate (SBN) mounted inside a quasi-self-imaging ring cavity. By changing the ratio of the intensities of the pump beams with a filter F of variable optical density, one can change the contributions of the laser-type gain due to the two-wave-mixing and the parametric-type gain due to the four-wave-mixing. The Fresnel number of the oscillator cavity is selected by positioning properly the intra-cavity lenses which are arranged in a nearly self-imaging configuration [9].

As expected, we have observed the vortices and the domain walls in the output beam for the two extreme cases when F has either minimal (0) or maximal (1) transmissions. At the intermediate F transmissions (~ 0.2), we have recorded an interferogram of the output field

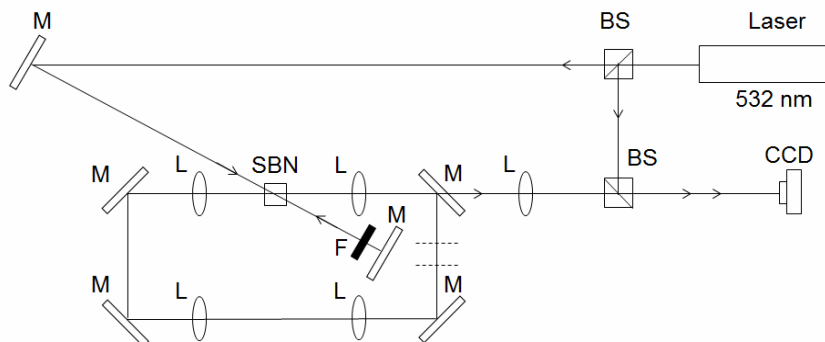


Fig. 6. A scheme of wave-mixing ring oscillator used in our experiments. M: mirrors, BS: beam splitters, L: lenses, SBN: photorefractive strontium-barium niobate crystal, F: variable-density filter, and CCD: charge-coupled device (camera).

structure shown in Fig. 7a. The insert in Fig. 7a displays a ‘fork’ located in the central part of the interferogram and the smooth shifts in its upper and lower parts, which are oppositely directed. Therefore the interferogram readily indicates that there is a vortex-like phase structure in the centre and Bloch-type domain wall structures with the opposite phase gradients in the adjacent sections. The structure rotates as shown in Fig. 7a. Processing the interferogram with a Fourier filtering method, we have obtained the appropriate spatial distributions of the intensity (see Fig. 7b) and phase (see Fig. 7c). They confirm that the point topological defect present in the centre of the structure is the ND.

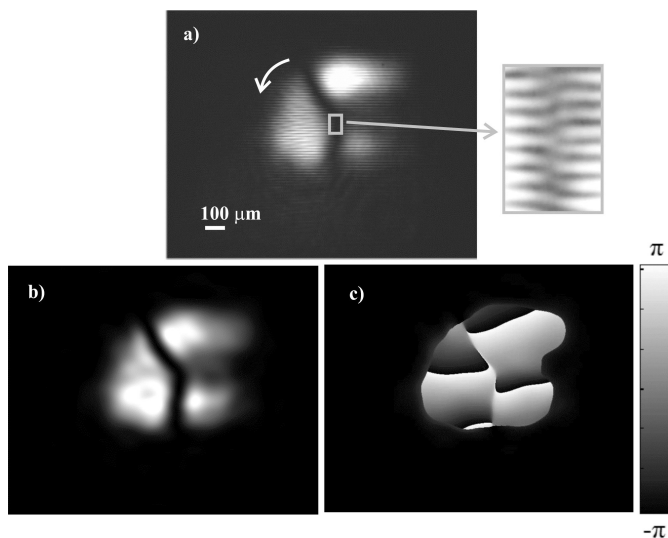


Fig. 7. Interferogram of the output beam (a), and reconstructed transverse spatial distributions of the intensity (b) and the phase (c).

4. Conclusion

The results reported in this study demonstrate that the optical oscillator with the mixed parametric and laser gains can generate composite optical topological objects build from the Bloch-type domain walls and the topological NDs, which reveal a spontaneous (translational or/and rotational) motion along the direction given by their own transverse phase gradients.

References

1. Nye J and Berry M, 1974. Dislocations in wave trains. *Proc. R. Soc. Lond. A.* **336**: 165–190.
2. Couillet P, Gil L and Rocca F, 1989. Optical vortices. *Opt. Commun.* **73**: 403–408.
3. Brambilla M, Battipede F, Lugiato L, Penna V, Prati F, Tamm C and Weiss C, 1991. Transverse laser patterns. I. Phase singularity crystals. *Phys. Rev. A.* **43**: 5090–5113.
4. Arecchi F, Giacomelli G, Ramazza P and Residori S, 1991. Vortices and defect statistics in two-dimensional optical chaos. *Phys. Rev. Lett.* **67**: 3749–3752.
5. Weiss C, Vaupel M, Staliunas K, Slekyas G and Taranenko V, 1999. Vortices and solitons in lasers. *Appl. Phys. B.* **68**: 151–168.
6. Trillo S, Haelterman M and Sheppard A, 1997. Stable topological spatial solitons in optical parametric oscillators. *Opt. Lett.* **22**: 970–972.
7. Staliunas K and Sanchez-Morcillo V, 1998. Spatial-localized structures in degenerate optical parametric oscillators. *Phys. Rev. A.* **57**: 1454–1457.
8. Taranenko V, Staliunas K and Weiss C, 1998. Pattern formation and localized structures in degenerated optical parametric mixing. *Phys. Rev. Lett.* **81**: 2236–2239.
9. Esteban-Martín A, Taranenko V, García J, Valcárcel G and Roldán E, 2005. Controlled

- observation of a nonequilibrium Ising–Bloch transition in a nonlinear optical cavity. Phys. Rev. Lett. **94**: 223903/1–4.
10. Valcárcel G and Staliunas K, 2003. Excitation of phase patterns and spatial solitons via two-frequency forcing of a 1:1 resonance. Phys. Rev. E. **67**: 026604/1–4.
 11. Esteban-Martín A, Martínez-Quesada M, Taranenko V, Roldán E and Valcárcel G, 2006. Bistable phase locking of a nonlinear optical cavity via rocking: Transmuting vortices into phase patterns. Phys. Rev. Lett. **97**: 093903/1–4.
 12. Kawagishi T, Mizugushi T and Sano M, 1995. Points, walls, and loops in resonant oscillatory media. Phys. Rev. Lett. **75**: 3768–3771.

Yaparov V. V. and Taranenko V. B. 2015. Moving domain walls with Néel defects in optical oscillator. Ukr.J.Phys.Opt. **16**: 159 – 164.

***Анотація.** На основі обчислень та експериментів показано, що оптичний параметричний генератор з комбінованим параметричним і лазерним підсиленням підтримує формування трансляційно або/і обертально рухомих композитних топологічних структур у формі оптичних доменних стінок з точковими дефектами, подібними до неєлівських топологічних дефектів у феромагнетиках і рідких кристалах.*