On the behaviour of Poynting vector in material media with weak optical activity

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Abstract. We have obtained phenomenological relation for the Poynting vector of electromagnetic wave propagating in crystals that possess a so-called weak optical activity. The appearance of transverse component of the Poynting vector and a transverse shift of the optical beam due to spin-orbit interaction are discussed.

Keywords: Poynting vector, weak optical activity

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1. Introduction

It is well known that optical activity is described by accounting for inhomogeneity of electric induction D_j appearing in the relation for electric field E_i of an optical wave that propagates through a medium:

$$E_i = B_{ij}^0 D_j + \gamma_{ijk} \frac{\partial D_j}{\partial x_k}, \quad (1)$$

where B_{ij}^0 denotes the optical-frequency impermeability tensor, γ_{ijk} the third-rank antisymmetric polar tensor ($\gamma_{ijk} = -\gamma_{jik}$), and x_k the coordinate. Using the known duality relation,

$$\frac{2\pi}{\lambda}\gamma_{ijk} = \delta_{ijl} g_{lk}, \quad (2)$$

one can reduce the tensor γ_{ijk} to a (generally nonsymmetric) second-rank axial gyration tensor g_{lk} (with δ_{ijl} being the unit antisymmetric Levi-Civita tensor). Then Eq. (1) may be rewritten as

$$E_i = (B_{ij}^0 + i\delta_{ijl} g_{lk} m_k) D_j, \quad (3)$$

where $k_k = \frac{2\pi}{\lambda} m_k$ denotes the wave vector of light and m_k the unit vector parallel to k_k. In its turn, the nonsymmetric gyration tensor can be decomposed into symmetric and antisymmetric parts:

$$g_{lk} = g_{lk}^s + g_{lk}^{as}, \quad (4)$$

A common point of view is that the optical activity effect is purely associated with the symmetric part of the gyration tensor. Scarce experimental data is available on manifestations of a kind of optical activity concerned with the antisymmetric part of the gyration tensor which is dual to some polar vector h_r,

$$g_{lk}^{as} = \delta_{lkr} h_r \,, \tag{5}$$

while the results of relevant theoretical analysis are very poor [1–5].

Recently, we have shown that this effect known as a weak optical activity should manifest itself in some changes of refractive indices and optical birefringence [6]. Moreover, if both the common optical activity and the weak one are simultaneously present, the polarisation state of one of the eigenwaves in crystal acquires a complicated longitudinally-transverse elliptical polarisation [7]. This novel polarisation eigenstate comprises two elliptical states, one of which is transversely elliptical and the other longitudinally elliptical.

Due to the Neumann's symmetry principle, the point symmetry groups for which $h_r \neq 0$ are 1, 2, *m*, *mm*2, 3, 3*m*, 4, 4*mm*, 6, 6*mm*, ∞, and ∞mm. Among these groups, only four reveal no common optical activity ($g_{lk}^{s} = 0$): 3*m*, 4*mm*, 6*mm*, and ∞mm. Eq. (3) for these media may be written as

$$E_i = \left(B_{ij}^0 + i\delta_{ijl}\delta_{lkr} h_r m_k \right) D_j \,. \tag{6}$$

It is obvious that only a single component of the h_r vector remains nonzero for the crystals that belong to the point groups 3*m*, 4*mm*, 6*mm* (namely, we have $h_3 \neq 0$).

2. Results of analysis

Let a plane electromagnetic wave propagate through a transparent, anisotropic, magnetically non-ordered, though spatially dispersive, medium. Consider the Poynting vector represented in the usual form [8, 9]

$$S_k^0 + S_k^1 = u_k W \,, \tag{7}$$

where $u_k W$ is the total electromagnetic energy flow, W the energy density, S_k^0 the energy flow without accounting for the spatial dispersion effect, S_k^1 the extra energy flow that appears if the spatial dispersion is present, $u_k = \partial\omega / \partial k_k$ means the group velocity, and ω the frequency of wave. The relations for the energy density and the time-averaged Poynting vector components are given by

$$W = \frac{1}{4}\left\{ \varepsilon_0 \frac{\partial\left(\omega\varepsilon_{ij}\right)}{\partial\omega} E_{0j} E_{0i}^{*} + \frac{1}{\mu_0} B_{0i} B_{0i}^{*} \right\}, \tag{8}$$

$$S_k^0 = \frac{1}{4\mu_0}\left\{ \left[E_{0j} \times B_{0i}^{*} \right] + \left[E_{0i}^{*} \times B_{0j} \right] \right\}, \tag{9}$$

$$S_k^1 = -\frac{1}{4}\varepsilon_0 \omega \left.\frac{\partial\varepsilon_{ij}}{\partial k_k}\right|_{\omega=\text{const}} E_{0j} E_{0i}^{*} \,, \tag{10}$$

where ε_{ij} stands for the dielectric permittivity, ε_0 the dielectric permittivity of vacuum, E_{0j}, E_{0i}^{*} the amplitudes of electric field components, B_0, B_0^{*} the amplitudes of the magnetic induction, and μ_0 the magnetic constant. When specifying Eq. (10), one needs equations for the dielectric permittivity tensor components and their wave-vector derivatives for the media that possess the weak optical activity.

Let us consider a particular case of optically uniaxial crystals, for which the following conditions are satisfied:

$$h_3 \neq 0;\ h_1 = h_2 = 0;\ B_{11}^0 = B_{22}^0 \neq B_{33}^0;\ m_2 = m_3 = 0;\ m_1 = 1\,. \tag{11}$$

The matrix that couples the components of the electric field and electric displacement vectors becomes

$$\begin{array}{c|ccc} & D_1 & D_2 & D_3 \\ \hline E_1 & B_{11}^0 & 0 & ih_3m_1 \\ E_2 & 0 & B_{11}^0 & 0 \\ E_3 & -ih_3m_1 & 0 & B_{33}^0 \end{array}, \tag{12}$$

so that we get a system of equations

$$\begin{cases} E_1 = B_{11}^0 D_1 + ih_3m_1D_3 \\ E_2 = B_{11}^0 D_2 \\ E_3 = -ih_3m_1D_1 + B_{33}^0 D_3 \end{cases}. \tag{13}$$

The corresponding relations that describe the electric induction of electromagnetic wave as a function of its electric field may be found as

$$\begin{cases} D_1 = \dfrac{B_{33}^0}{B_{11}^0B_{33}^0 - (h_3m_1)^2}E_1 - i\dfrac{h_3m_1}{B_{11}^0B_{33}^0 - (h_3m_1)^2}E_3 \\ D_2 = \dfrac{1}{B_{11}^0}E_2 \\ D_3 = i\dfrac{h_3m_1}{B_{11}^0B_{33}^0 - (h_3m_1)^2}E_1 + \left(\dfrac{1}{B_{33}^0} + \dfrac{(h_3m_1)^2}{B_{33}^0\left(B_{11}^0B_{33}^0 - (h_3m_1)^2\right)}\right)E_3 \end{cases}. \tag{14}$$

Thus, the dielectric permittivity tensor is as follows:

$$\varepsilon_{ij} = \begin{vmatrix} \dfrac{B_{33}^0}{B_{11}^0B_{33}^0 - (h_3m_1)^2} & 0 & -\dfrac{ih_3m_1}{B_{11}^0B_{33}^0 - (h_3m_1)^2} \\ 0 & \dfrac{1}{B_{11}^0} & 0 \\ \dfrac{ih_3m_1}{B_{11}^0B_{33}^0 - (h_3m_1)^2} & 0 & \dfrac{1}{B_{33}^0} + \dfrac{(h_3m_1)^2}{B_{33}^0\left(B_{11}^0B_{33}^0 - (h_3m_1)^2\right)} \end{vmatrix}. \tag{15}$$

Taking into account the relation $h_3m_1 = h_3k_1\lambda/2\pi = h_3k_1c/\omega$, one can rewrite the matrix given by Eq. (15) as

$$\varepsilon_{ij} = \begin{vmatrix} \dfrac{\omega^2B_{33}^0}{\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2} & 0 & -i\dfrac{\omega h_3k_1c}{\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2} \\ 0 & \dfrac{1}{B_{11}^0} & 0 \\ i\dfrac{\omega h_3k_1c}{\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2} & 0 & \dfrac{1}{B_{33}^0}\left(1 + \dfrac{(h_3k_1c)^2}{\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2}\right) \end{vmatrix}. \tag{16}$$

Then the wave-vector derivatives of the dielectric tensor components reduce to

$$\left.\frac{\partial(\varepsilon_{11})}{\partial k_1}\right|_{\omega=\mathrm{const}} = \frac{2k_1\omega^2 B_{33}^0 (h_3 c)^2}{\left[\omega^2 B_{11}^0 B_{33}^0 - (h_3 k_1 c)^2\right]^2}, \tag{17}$$

$$\left.\frac{\partial(\varepsilon_{22})}{\partial k_1}\right|_{\omega=\mathrm{const}} = 0, \tag{18}$$

$$\left.\frac{\partial(\varepsilon_{33})}{\partial k_1}\right|_{\omega=\mathrm{const}} = \frac{2k_1\omega^2 (h_3 c)^2 B_{11}^0}{\left[\omega^2 B_{11}^0 B_{33}^0 - (h_3 k_1 c)^2\right]^2}, \tag{19}$$

$$\left.\frac{\partial(\varepsilon_{13})}{\partial k_1}\right|_{\omega=\mathrm{const}} = -i\omega h_3 c \frac{\omega^2 B_{11}^0 B_{33}^0 + (h_3 k_1 c)^2}{\left[\omega^2 B_{11}^0 B_{33}^0 - (h_3 k_1 c)^2\right]^2}, \tag{20}$$

Using Eqs. (17)–(20), one can represent Eq. (10) as

$$\begin{aligned} S_1^1 = &-\frac{1}{2}\varepsilon_0 \frac{k_1\omega^3 (h_3 c)^2}{\left[\omega^2 B_{11}^0 B_{33}^0 - (h_3 k_1 c)^2\right]^2}\left(B_{33}^0 E_{01} E_{01}^* + B_{11}^0 E_{03} E_{03}^*\right) \\ &+\frac{1}{4} i\varepsilon_0 \omega^2 h_3 c \frac{\omega^2 B_{11}^0 B_{33}^0 + (h_3 k_1 c)^2}{\left[\omega^2 B_{11}^0 B_{33}^0 - (h_3 k_1 c)^2\right]^2}\left(E_{03} E_{01}^* - E_{01} E_{03}^*\right) \end{aligned}. \tag{21}$$

In the assumptions that E_{01}^* and E_{03}^* are real, Eq. (21) for the component S_1^1 of the Poynting vector consists only of real part. The real part appearing in Eq. (21) is associated with some changes in the group velocity of electromagnetic wave caused by the weak optical activity.

As follows from Eqs. (14), the component D_1 is a pure result of the weak optical activity. Due to the condition $D_1 \perp B_2 \perp k_3$, the component k_3 of the wave vector has to arise, too. This means that Eqs. (17)–(20) should also include the derivatives of k_3, thus inevitably leading to appearance of the component S_3^1 of the Poynting vector. In fact this implies that the light beam should ‘drift’ along *Z* direction while propagating in crystals with the weak optical activity. This effect can be easily explained when taking into account that the elliptically polarised photon, with its polarisation ellipse belonging to *XZ* plane, should possess a spin-orbit momentum component s_2. The existence of this component of spin angular momentum would lead to the drift of photon mentioned above, as a result of a spin-orbit interaction and an optical Magnus effect [10–12].

Now let us derive the relation for the S_3^1 component of the Poynting vector. As already mentioned, the weak optical activity leads to inclination of the $\vec{D}$ component of electric induction of the electromagnetic wave propagating along *X* direction, with appearance of D_1 component. The ratio of these components can be expressed as

$$\frac{D_1}{D_3} = \frac{B_{33}^0 E_1 - ih_3 m_1 E_3}{ih_3 m_1 E_1 + B_{11}^0 E_3} = -\frac{k_3}{k_1}. \tag{22}$$

Taking into account that $m_1 = k_1 c/\omega$, one can rewrite Eq. (22) in the form of quadratic equation with respect to the wave vector k_1:

$$ih_3 (k_1)^2 cE_3 - k_1 E_1\left(\omega B_{33}^0 + ih_3 k_3 c\right) - \omega B_{11}^0 k_3 E_3 = 0, \tag{23}$$

with the solutions

$$(k_1)_{1,2} = \frac{1}{2ih_3cE_3} \times\left[\omega B_{33}^0 E_1 + ih_3k_3cE_1 \pm \sqrt{\omega^2\left(B_{33}^0\right)^2 E_1^2 - (k_3)^2(h_3c)^2 E_1^2 + 2ih_3c\omega k_3\left(B_{33}^0E_1^2 + 2B_{11}^0E_3^2\right)}\right], \quad (24)$$

of which derivatives can be represented as

$$\left.\frac{\partial k_1}{\partial k_3}\right|_{\omega=\text{const}} = \frac{E_1}{2E_3} \pm \frac{\omega\left(B_{33}^0E_1^2 + 2B_{11}^0E_3^2\right) + ik_3h_3cE_1^2}{2E_3\sqrt{\omega^2\left(B_{33}^0\right)^2 E_1^2 - (k_3)^2(h_3c)^2 E_1^2 + 2ih_3c\omega k_3\left(B_{33}^0E_1^2 + 2B_{11}^0E_3^2\right)}}. \quad (25)$$

Considering Eq. (22), one can rewrite Eq. (25) as

$$\left.\frac{\partial k_1}{\partial k_3}\right|_{\omega=\text{const}} = \frac{E_1}{2E_3} \pm \frac{\left(\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2\right)E_1^2 + 2ih_3k_1c\omega B_{11}^0E_1E_3 + 2\left(\omega B_{11}^0\right)^2E_3^2}{2\left(\omega^2B_{11}^0B_{33}^0 + (h_3k_1c)^2\right)E_1E_3 - 4ih_3k_1c\omega B_{11}^0E_3^2}. \quad (26)$$

Since the S_3^1 vector is defined by the relation

$$S_3^1 = -\frac{1}{4}\varepsilon_0\omega\left.\frac{\partial\varepsilon_{ij}^{/}}{\partial k_3}\right|_{\omega=\text{const}} E_{0j}E_{0i}^* = -\frac{1}{4}\varepsilon_0\omega\left.\frac{\partial\varepsilon_{ij}^{/}}{\partial k_1}\right|_{\omega=\text{const}}\left.\frac{\partial k_1}{\partial k_3}\right|_{\omega=\text{const}} E_{0j}E_{0i}^* = \left.\frac{\partial k_1}{\partial k_3}\right|_{\omega=\text{const}} S_1^1, \quad (27)$$

one can write out the final expression for the Z component of the Poynting vector:

$$S_3^1 = -\frac{\varepsilon_0(h_3c)^2\omega^3k_1\left(B_{33}^0E_{01}E_{01}^* + B_{11}^0E_{03}E_{03}^*\right)}{4\left[\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2\right]^2} \times\left[\frac{E_1}{E_3} \pm \frac{\left(\left(\omega^2B_{11}^0B_{33}^0\right)^2 - (h_3k_1c)^4\right)E_1^3E_3 + 2\left(\omega B_{11}^0\right)^2\left(\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2\right)E_1E_3^3}{\left(\omega^2B_{11}^0B_{33}^0 + (h_3k_1c)^2\right)^2E_1^2E_3^2 + 4\left(h_3k_1c\omega B_{11}^0\right)^2E_3^4}\right]$$
$$-\frac{\varepsilon_0(h_3c)^2\omega^3k_1\left(B_{33}^0E_{01}E_{01}^* + B_{11}^0E_{03}E_{03}^*\right)}{4\left[\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2\right]^2} \times \frac{4ih_3k_1c\omega^3\left(B_{11}^0\right)^2\left[B_{33}^0E_1^2E_3^2 + B_{11}^0E_3^4\right]}{\left(\omega^2B_{11}^0B_{33}^0 + (h_3k_1c)^2\right)^2E_1^2E_3^2 + 4\left(h_3k_1c\omega B_{11}^0\right)^2E_3^4}. \quad (28)$$

In the assumptions $E_{01} = E_{01}^* = E_1$ and $E_{03} = E_{03}^* = E_3$, Eq. (28) reads as

$$S_3^1 = S_1^1 \times \left.\frac{\partial k_1}{\partial k_3}\right|_{\omega=\text{const}} == -\frac{\varepsilon_0(h_3)^2k_1c^2\omega^3\left(B_{33}^0E_1^2 + B_{11}^0E_3^2\right)}{4\left[\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2\right]^2} \times$$
$$\left[\frac{E_1}{E_3} \pm \frac{\left(\left(\omega^2B_{11}^0B_{33}^0\right)^2 - (h_3k_1c)^4\right)E_1^3E_3 + 2\left(\omega B_{11}^0\right)^2\left(\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2\right)E_1E_3^3}{\left(\omega^2B_{11}^0B_{33}^0 + (h_3k_1c)^2\right)^2E_1^2E_3^2 + 4\left(h_3k_1c\omega B_{11}^0\right)^2E_3^4}\right]$$
$$-\frac{\varepsilon_0(h_3)^2k_1c^2\omega^3\left(B_{33}^0E_1^2 + B_{11}^0E_3^2\right)}{4\left[\omega^2B_{11}^0B_{33}^0 - (h_3k_1c)^2\right]^2} \times \frac{4ih_3k_1c\omega^3\left(B_{11}^0\right)^2\left[B_{33}^0E_1^2E_3^2 + B_{11}^0E_3^4\right]}{\left(\omega^2B_{11}^0B_{33}^0 + (h_3k_1c)^2\right)^2E_1^2E_3^2 + 4\left(h_3k_1c\omega B_{11}^0\right)^2E_3^4}. \quad (29)$$

Now one can extract the real part of the Z component of the Poynting vector basing on Eq. (29). It is equal to

$$\left(S_3^1\right)_{\text{Re}} = -\frac{\varepsilon_0 \left(h_3\right)^2 k_1 c^2 \omega^3 \left(B_{33}^0 E_1^2 + B_{11}^0 E_3^2\right)}{4\left[\omega^2 B_{11}^0 B_{33}^0 - \left(h_3 k_1 c\right)^2\right]^2}$$
$$\times\left[\frac{E_1}{E_3} \pm \frac{\left(\left(\omega^2 B_{11}^0 B_{33}^0\right)^2 - \left(h_3 k_1 c\right)^4\right) E_1^3 E_3 + 2\left(\omega B_{11}^0\right)^2 \left(\omega^2 B_{11}^0 B_{33}^0 - \left(h_3 k_1 c\right)^2\right) E_1 E_3^3}{\left(\omega^2 B_{11}^0 B_{33}^0 + \left(h_3 k_1 c\right)^2\right)^2 E_1^2 E_3^2 + 4\left(h_3 k_1 c \omega B_{11}^0\right)^2 E_3^4}\right], \tag{30}$$

or

$$\left(S_3^1\right)_{\text{Re}} = -\frac{\varepsilon_0 \left(h_3\right)^2 k_1 c^2 \omega^3 \left(B_{33}^0 E_1^2 + B_{11}^0 E_3^2\right)}{4\left[\omega^2 B_{11}^0 B_{33}^0 - \left(h_3 k_1 c\right)^2\right]^2} \frac{E_1}{E_3}$$
$$\mp \frac{\varepsilon_0 \left(h_3\right)^2 k_1 c^2 \omega^3 \left(B_{33}^0 E_1^2 + B_{11}^0 E_3^2\right)}{4\left[\omega^2 B_{11}^0 B_{33}^0 - \left(h_3 k_1 c\right)^2\right]^2} \frac{\left(\left(\omega^2 B_{11}^0 B_{33}^0\right)^2 - \left(h_3 k_1 c\right)^4\right) E_1^3 E_3}{\left(\omega^2 B_{11}^0 B_{33}^0 + \left(h_3 k_1 c\right)^2\right)^2 E_1^2 E_3^2 + 4\left(h_3 k_1 c \omega B_{11}^0\right)^2 E_3^4} \tag{31}$$
$$\mp \frac{\varepsilon_0 \left(h_3\right)^2 k_1 c^2 \omega^3 \left(B_{33}^0 E_1^2 + B_{11}^0 E_3^2\right)}{4\left[\omega^2 B_{11}^0 B_{33}^0 - \left(h_3 k_1 c\right)^2\right]^2} \frac{2\left(\omega B_{11}^0\right)^2 \left(\omega^2 B_{11}^0 B_{33}^0 - \left(h_3 k_1 c\right)^2\right) E_1 E_3^3}{\left(\omega^2 B_{11}^0 B_{33}^0 + \left(h_3 k_1 c\right)^2\right)^2 E_1^2 E_3^2 + 4\left(h_3 k_1 c \omega B_{11}^0\right)^2 E_3^4}.$$

Neglecting the smallest second and third terms in the r. h. s. of Eq. (31), one can represent this equation in the following form:

$$\left(S_3^1\right)_{\text{Re}} = -\frac{\varepsilon_0 \left(h_3\right)^2 k_1 c^2 \omega^3 \left(B_{33}^0 E_1^2 + B_{11}^0 E_3^2\right)}{4\left[\omega^2 B_{11}^0 B_{33}^0 - \left(h_3 k_1 c\right)^2\right]^2} \frac{E_1}{E_3}$$
$$= -\frac{\varepsilon_0 \left(h_3\right)^2 k_1 c^2 \omega^3 \left(B_{33}^0 E_1^2 + B_{11}^0 E_3^2\right)}{4\omega^4 {B_{11}^0}^2 {B_{33}^0}^2 - 8\omega^2 B_{11}^0 B_{33}^0 \left(h_3 k_1 c\right)^2 + 4\left(h_3 k_1 c\right)^4} \frac{E_1}{E_3}. \tag{32}$$

When accounting that $-8\omega^2 B_{11}^0 B_{33}^0 \left(h_3 k_1 c\right)^2 + 4\left(h_3 k_1 c\right)^4 << 4\omega^4 \left(B_{11}^0 B_{33}^0\right)^2$, one gets

$$\left(S_3^1\right)_{\text{Re}} \approx -\frac{\varepsilon_0 \left(h_3\right)^2 k_1 c^2 \left(B_{33}^0 E_1^2 + B_{11}^0 E_3^2\right)}{4\omega {B_{11}^0}^2 {B_{33}^0}^2} \frac{E_1}{E_3} = \left(S_1^1\right)_{\text{Re}} \frac{E_1}{E_3}. \tag{33}$$

Finally, if a small optical anisotropy is assumed ($B_{33}^0 \simeq B_{11}^0$), Eq. (33) reduces to

$$\left(S_3^1\right)_{\text{Re}} \approx -\frac{1}{4} \bar{n}^7 \varepsilon_0 \left(h_3\right)^2 c\, I_{tot} \frac{E_1}{E_3} = \left(S_1^1\right)_{\text{Re}} \frac{E_1}{E_3}, \tag{34}$$

where $\bar{n}$ is the mean refractive index and I_{tot} the total light intensity propagating in crystal. As seen from Eqs. (33) and (34), the Z component of the real part of the Poynting vector does not depend on the sign of vector h_3, though it is dependent on the module of the latter, as well as the total intensity of light, the refractive index, and the ratio E_1 / E_3. Notice that the condition $E_1 << E_3$ gives rise to the relation $\left(S_3^1\right)_{\text{Re}} << \left(S_1^1\right)_{\text{Re}}$.

3. Conclusions

In the present work we have obtained a relation for the Poynting vector of electromagnetic wave that propagates in crystals possessing a so-called weak optical activity. It has been revealed that the Poynting vector has some transverse component that describes transverse shift of the optical beam. Such a beam drift could be explained following from the quantum properties of photon. Namely, longitudinal elliptical polarisation of electromagnetic wave caused by the weak optical activity corresponds to a transverse spin of photon. Due to spin-orbit interaction, the transverse spin results in the transverse drift of that photon.

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Анотація. *Одержано співвідношення для вектора Пойнтінґа електромагнітної хвилі, яка поширюється в кристалах зі слабкою оптичною активністю. У роботі обговорено появу поперечної компоненти вектора Пойнтінґа та поперечного зміщення оптичного променя внаслідок спін-орбітальної взаємодії.*