Measurements of piezooptic coefficients $\pi_{14}$ and $\pi_{25}$ in $\text{Pb}_5\text{Ge}_3\text{O}_{11}$ crystals using torsion induced optical vortex

Vasylkiv Yu., Skab I. and Vlokh R.

Institute of Physical Optics, 23 Dragomanov St., 79005 Lviv, Ukraine, vlokh@ifo.lviv.ua

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Abstract

We show how a singularity of optical field can be used for determination of piezooptic coefficient of crystals under their torsion, in particular in the situation when the natural optical activity makes direct polarimetric experiments too complicated. Experimental studies of an optical vortex appearing under torsion of lead germanate crystals have enabled us to determine so-called non-principal coefficients ($\pi_{14}$ and $\pi_{25}$). A relationship between the spin angular momentum of light beam and the piezooptic coefficients has been derived.

Keywords: torsion stress, optical vortex, spin angular momentum, piezooptic coefficients, lead germanate crystals

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1. Introduction

Recently we have suggested a new high-accuracy torsion method for studies of piezooptic effect in solids [1–5]. As shown in our works, the method generally provides fairly high accuracy for piezooptic coefficients. In particular, the error for so-called non-principal coefficients has been reduced to a few percents [2, 3] (Concerning the piezooptic effect itself, we refer reader to Ref. [6]). As a result, we have successfully determined the piezooptic coefficient $\pi_{14}$ for $\text{LiNbO}_3$ crystals belonging to the point symmetry group $3\text{m}$ and $\alpha\text{-BaB}_2\text{O}_4$ crystal (the point group $\overline{3}\text{m}$) [2–4]. It has been demonstrated that the sign of the piezooptic coefficients, e.g. the coefficient $\pi_{14}$, can also be determined using the torsion method [4].

The authors have found that the method mentioned above enables measuring, easily enough, the piezooptic coefficients for the crystals whose symmetry groups include a three-fold symmetry axis, i.e. the groups $3\text{m}$, $\overline{3}\text{m}$, 3, $\overline{3}$, 32, 432, m3, 23, and $\overline{4}\text{3}m$ [3, 5, 7]. Let us notice that, in the case of cubic crystals, one can determine a combination of the coefficients $\pi_{11}$, $\pi_{12}$, $\pi_{13}$ and $\pi_{44}$ under torsion torque applied around $\{111\}$ direction (see [7]). For the crystals of 3 and $\overline{3}$ symmetry groups, the coefficients $\pi_{14}$ and $\pi_{25}$ are measurable when the torsion is applied around the optic axis and the light propagates along the same direction [5]. The experimental geometries needed for crystals of the other symmetries remain more complicated.
It is worthwhile that, due to a lack of inversion symmetry, some of the trigonal or cubic crystals (namely, those belonging to the point groups 3, 32, 432, and 23) permit a natural optical activity. Then the optical eigenwaves propagating along the three-fold axis would become elliptically polarised. Moreover, a circular birefringence associated with the optical activity can even turn out to be higher than its linear counterpart induced by the torsion stresses due to the piezooptic effect. In this case the relations for the quantities determined experimentally, e.g. for the azimuth of polarisation ellipse of the emergent light and the ellipticity of the latter, become nonlinear and very cumbersome [8–10]. As a consequence, one has very serious difficulties in finding unique solutions of appropriate systems of equations for the induced linear birefringence and the orientation of optical indicatrix. This would lead to inevitable difficulties with determination of the piezooptic coefficients.

Nonetheless, it is known [5, 7, 11] that the torsion of crystals belonging to the both trigonal and cubic symmetry groups is accompanied by appearance of an optical vortex, whenever the crystal sample is placed between circular polarisers. Then the intensity distribution in a wide cross section of a nearly plane-wave optical beam transmitted through the output circular polariser is very peculiar: the intensity remains equal to zero in the beam centre and a bright ring is observed around a central dark region.

In the present work we will show that the latter peculiarity could, in principle, be used for determining the piezooptic coefficients of optically active trigonal crystals. This will be done on the example of lead germanate crystals, Pb₅Ge₃O₁₁, belonging to the point symmetry group 3 under normal conditions [12–14]. Moreover, as far as we know the piezooptic coefficients π₁₄ and π₂₅ for these crystals have not yet been reported in the literature.

2. Basic relations, experimental procedure, and results

If the crystals under analysis are twisted, the distribution of the shear stress components σ₄ and σ₅ around Z axis may be written as [15]

$$\sigma_4 = \frac{2M_Z}{\pi R^4} X,$$

$$\sigma_5 = \frac{2M_Z}{\pi R^4} Y,$$

where $M_Z$ denotes the torque moment, $R$ the radius of cylindrical crystalline rod, and the generatrix line is parallel to Z, with X, Y and Z being the principal axes of the Fresnel ellipsoid. As shown in the study [5], the spatial distribution of the birefringence in the XY plane for the crystals belonging to the group 3 is determined by the relation

$$\Delta n_{XY} = n_0^3 \left[ \pi_4^2 \sigma_4^2 + (\pi_{25} \sigma_4 + \pi_{14} \sigma_5)^2 \right] = 2n_0^3 \frac{M_Z}{\pi R^4} \sqrt{\pi_4^2 X^2 + (\pi_{25} X + \pi_{14} Y)^2},$$

while the angle of optical indicatrix rotation around the Z axis may be represented as
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\[
\tan 2Z = \frac{\sigma_25\sigma_4 + \sigma_14\sigma_5}{\sigma_14\sigma_4} = \frac{\pi_25X + \pi_14Y}{\pi_14X}.
\]  

(4)

As one can easily see, Eq. (3) represents equation of an elliptical cone. Then the spatial distribution of the light intensity behind the quarter-wave plate in the system 'right-handed polariser, crystalline sample, left-handed polariser' (see Fig. 1) should be written in the following form:

\[
I = I_0 \sin^2 \frac{\Delta\Gamma}{2} \cos^2 2\epsilon,
\]

(5)

where

\[
\Delta\Gamma^2 = \Delta\Gamma_L^2 + \Delta\Gamma_C^2 = \left(\frac{2\pi\Delta n_{XY} d_Z}{\lambda}\right)^2 + (2\rho d_Z)^2
\]

(6)

is the total phase difference, \(\Delta\Gamma_L\) and \(\Delta\Gamma_C\) the partial phase differences caused respectively by the linear (\(\Delta n_L\)) and circular (\(\Delta n_C\)) birefringences, \(d_Z = 14.5\) mm the sample thickness along the direction of light propagation, \(\rho = 97\) rad/m the specific rotation of polarisation plane, and \(\epsilon\) the ellipticity angle of the eigenwaves defined by the relation

\[
tg\epsilon = \frac{1}{g_{33}n_0}\left(n_0\Delta n_{XY} - \sqrt{4n_0^2\Delta n_{XY}^2 + g_{33}^2n_0^2}\right).
\]

(7)

Here \(\lambda = 632.8\) nm is the light wavelength, \(R = 2.55\) mm the radius of cross section of the sample, \(M_Z = 0.248\) N m the torsion moment, \(g_3 = 4.02 \times 10^{-5}\) and \(n_0 = 2.115\) respectively the gyration tensor component and the ordinary refractive index [9].

![Fig. 1. Experimental setup: 1 – He-Ne laser, 2 – incident light intensity adjuster, 3 – coherence scrambler, 4 – beam expander, 5 – linear polariser, 5 – right-handed circular polariser, 6 – sample, 7 – left-handed circular polariser, 8 – objective lens, 9 – CCD camera, 10 – controller of step motors, 11 – personal computer.](image-url)
The experimental procedure is as follows. A wide incident (nearly plane-wave) beam with the linear polarisation shaped by a beam expander 4 propagates through an input right-handed circular polariser 5. It becomes circularly polarised and then propagates through a sample 6 and a next left-handed circular polariser 7. The intensity distribution is detected by a CCD camera 9. As shown in our recent works [7, 11], an optical vortex should appear behind an output circular polariser (7), which in general would have an elliptical intensity distribution and a topological charge equal to ±1. Here the optical vortex appears owing to a process similar to that observed in the case of conical refraction in biaxial crystals [16–18], while the elliptical intensity distribution is caused by a mixed screw-edge dislocation of the wave front [7, 19].

As seen from Eq. (3), the elliptical conical distribution of the induced birefringence would appear only when the magnitudes of \( \pi_{25} \) and \( \pi_{14} \) are comparable. Otherwise, when \( \pi_{25} \) is much smaller than \( \pi_{14} \) or equal to zero, Eq. (3) may be rewritten as

\[
\Delta n_{xy} = 2n_0^3 \frac{M_Z}{\pi R^4} \pi_{14} \sqrt{x^2 + y^2} = 2n_0^3 \frac{M_Z}{\pi R^4} \pi_{14} r,
\]

where the \( r \) coordinate refers to the polar system. Notice that Eq. (8) describes a canonical cone with a circular basis. In fact, we have observed an almost circular intensity distribution in the vortex beam obtained on the twisted Pb₅Ge₃O₁₁ crystal (see Fig. 2). This means that the wave front contains almost a pure screw dislocation, while the \( \pi_{25} \) coefficient for the Pb₅Ge₃O₁₁ crystals is negligibly small.

\[ Fig. 2. \] Intensity distribution in the optical vortex created using a twisted Pb₅Ge₃O₁₁ crystal (\( \lambda = 6.328 \text{ nm}, M_Z = 0.248 \text{ N×m} \)).

After fitting, along different diameters, the image presented in Fig. 2 with Eq. (5) and accounting for Eq. (8), one can evaluate the coefficient \( \pi_{14} \). As an example, Fig. 3 shows the intensity distribution obtained experimentally along the \( X \) axis and the result of
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its fitting. Following from the results obtained, we have determined the module of the piezooptic coefficient $\pi_{14}$ for the lead germanate: $|\pi_{14}| = 0.35 \times 10^{-12} \text{m}^2/\text{N}$.

![Graph showing intensity distribution along X axis.](image)

**Fig. 3.** Intensity distribution along X axis (see text): points denote the experimental data and solid curve the result of fitting with Eq. (5).

The contrast of the bright ring against the background of the central dark ring depends on the torque moment and the magnitude of the piezooptic coefficient. It increases when the mentioned quantities do. At the same time, the central dark spot becomes narrower (see [11]). Such a behaviour suggests that increasing torque moment (i.e., the piezooptic coefficient) induces decrease in the total spin angular momentum of the outgoing beam, which is given by the relation

$$ S_Z^{\text{out}} = (I_r^{\text{out}} - I_l^{\text{out}}) / (I_r^{\text{out}} + I_l^{\text{out}}) $$

(with $I_r$ and $I_l$ standing for the total intensities of the right-hand and left-hand polarised beam components, respectively) [11]. Hence, both the orbital angular momentum of the emergent beam and the mechanical momentum transferred to the sample could increase with increasing torque moment, owing to the momentum conservation law:

$$ L_Z^{\text{inc}} + S_Z^{\text{inc}} = L_Z^{\text{out}} + S_Z^{\text{out}} + M_Z^{\text{mech}}, \quad (9) $$

where $L_Z^{\text{inc}} = 0$ and $S_Z^{\text{inc}} = 1$ are respectively the orbital angular and spin angular momentums of the incident beam, $L_Z^{\text{out}}$ and $S_Z^{\text{out}}$ respectively the orbital angular and spin angular momentums of the outgoing beam, and $M_Z^{\text{mech}}$ the mechanical angular momentum transferred to the sample due to the Beth effect [20].

In our particular case, we rewrite Eq. (9) as

$$ S_Z^{\text{inc}} = M_Z^{\text{mech}} + L_Z^{\text{out}} + \frac{(I_r^{\text{out}} - I_l^{\text{out}})}{(I_r^{\text{out}} + I_l^{\text{out}})} = 1, \quad (10) $$

so that the spin angular momentum of the outgoing beam is expressed by
Using the experimental image (see Fig. 2), one can determine the $S^\text{out}$ values for different $X$ and $Y$ coordinates (Table 1). Then one can obtain the piezooptic coefficient $\pi_{14}$ basing on Eq. (11) and Eqs. (6)–(8).

Table 1. Modules of the piezooptic coefficient $\pi_{14}$ that correspond to different spin angular momentums $S^\text{out}_Z$ and different coordinates $X$ ($Y=0$).

| Coordinate $X$, mm | Spin angular momentum $S^\text{out}_Z$ | Piezooptic coefficient $|\pi_{14}|.10^{-12} \text{m}^2/\text{N}$ |
|-------------------|----------------------------------|----------------------------------|
| 0.7               | 0.58                             | 0.45                             |
| 1.0               | 0.76                             | 0.21                             |
| 1.5               | 0.20                             | 0.40                             |
| 1.7               | 0.17                             | 0.35                             |
| 2.0               | 0.37                             | 0.25                             |
| -0.5              | 0.80                             | 0.40                             |
| -1.0              | 0.67                             | 0.28                             |
| -1.5              | 0.49                             | 0.25                             |
| -1.7              | 0.36                             | 0.46                             |
| -2.0              | 0.54                             | 0.43                             |

Using the data presented in Table 1, we are now able to calculate the average value of the piezooptic coefficient and the error of its determination: $|\pi_{14}|=(0.35 \pm 0.09) \times 10^{-12} \text{m}^2/\text{N}$. It is evident that the error for the piezooptic coefficient is equal to 26%. Therefore the $\pi_{25}$ coefficient should be still smaller than that error ($|\pi_{25}| \leq 0.09 \times 10^{-12} \text{m}^2/\text{N}$). When compared to the relative piezooptic errors achieved earlier for the both cases of LiNbO$_3$ (3.1% [3]) and $\alpha$-BaB$_2$O$_4$ crystals (9.6% [4]), the error for the $\pi_{14}$ coefficient of Pb$_5$Ge$_3$O$_{11}$ is high enough. As already explained, this should be associated with the fact that the lead germanate crystals possess the natural optical activity which hinders precise direct polarimetric measurements of the optical retardation.

While observing the changes in the conoscopic fringes that appear when the light propagates through the twisted Pb$_5$Ge$_3$O$_{11}$ crystal (this method has been in detail described in our recent work [4]), we have determined the sign of the $\pi_{14}$ coefficient. Since the coefficient is negative, we have the final result $\pi_{14} = -(0.35 \pm 0.09) \times 10^{-12} \text{m}^2/\text{N}$.

3. Conclusion

In conclusion, we have shown that the singularity of optical wave front can be used in practice when determining so-called non-principal piezooptic coefficients. In the present work we have determined the piezooptic coefficient $\pi_{14}$ for optically active lead germa-
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tate crystals. This has been achieved by means of observation of the optical vortex appearing under crystal torsion. It has been found that piezooptic coefficient $\pi_{14}$ is equal to $\pi_{14} = -(0.35 \pm 0.09) \times 10^{-12} \text{m}^2/\text{N}$, while for the coefficient $\pi_{25}$ we have $|\pi_{25}| \leq 0.09 \times 10^{-12} \text{m}^2/\text{N}$.

References


Анотація. В роботі показано, яким чином можна використати сингулярності оптичного поля для вимірювання п’єзооптичних коефіцієнтів при крученні кристалів, за умови, що природна оптична активність створює перешкоди прямим поляриметричним дослідженням. Експериментальне дослідження оптичного вихору, який виникав при крученні кристалів германату свинцю дозволило визначити, так звані, неголовні п’єзооптичні коефіцієнти (π₁₄ та π₂₅). Отримано співвідношення між спіновим кутовим моментом пучка і п’єзооптичними коефіцієнтами.