Optical-Gravitation Nonlinearity: A Change of Gravitational Coefficient *G* induced by Gravitation Field

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Abstract

Nonlinear effect of the gravitation field of spherically symmetric mass on the gravitational coefficient *G* has been analysed. In frame of the approaches of parametric optics and gravitation nonlinearity we have shown that the gravitation field of spherically symmetric mass can lead to changes in the gravitational coefficient *G*.

Keywords: gravitation field, gravitational coefficient, optical-mechanical analogy in general relativity

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Introduction

Gravitational coefficient G is one of fundamental constants characterising nature. It is often used as a so-called "Okkam's blade", when checking reliability of new or modified theories of gravity. Thus, there arises a problem of determining precisely the absolute value of *G* and its possible variations with time and range. Surprisingly, disparity among the G values derived by different groups has been gradually increasing [1–5] with increasing precision of the measurements. It has led to 1998 CODATA decision concerned with the increase in the relative uncertainty of *G* from 0.013% up to 0.15% [6]. This situation has triggered several theoretical and experimental attempts to extricate the *G* problems mentioned above [7–10]. Here we propose considering possible variations of G in frame of the optical-mechanical analogy adopted within the general relativity [11,12] and an optical-gravitation nonlinearity. This approach has allowed us utilising successfully the approved techniques of the parametric optics for description of influence of the gravitation field on electromagnetic radiation [13–15].

Refractive index dependence on the gravitation field

According to [10], the coordinate dependence of the refractive index change in the gravitation field of spherical mass M may be written as

$$n(r)=\left(1+\frac{m}{2r}\right)^{3}\left(1-\frac{m}{2r}\right)^{-1}, \qquad (1),$$

where $m=\frac{GM}{c_0^2}$, c_0 is the light speed in vacuum and G the gravitation constant. Taking the relation $\frac{GM}{c_0^2}=\frac{r_{Sch}}{2}$ into account (with r_{Sch} being the Schwarzschild radius), we may rewrite Eq. (1) as

$$n=\left(1+\frac{r_{Sch}}{4r}\right)^{3}\left(1-\frac{r_{Sch}}{4r}\right)^{-1}. \qquad (2)$$

It is easy to see that the ratio $\frac{r_{Sch}}{4r}$ is usually much smaller than unity. Then one can construct a convergent power series expansion and transform Eq. (2) to

$$n = 1 + 4\frac{m}{2r} + 7\left(\frac{m}{2r}\right)^2 + 8\left(\frac{m}{2r}\right)^3 + \ldots \quad (3)$$

Let us take into account that $\frac{1}{r} = \sqrt{\frac{g}{GM}}$ (g being the gravitation field strength). Then Eq. (3) may be represented as

$$n = 1 + 4\frac{\sqrt{GM}}{2c_0^2}\sqrt{g} + 7\left(\frac{\sqrt{GM}}{2c_0^2}\right)^2\left(\sqrt{g}\right)^2 + 8\left(\frac{\sqrt{GM}}{2c_0^2}\right)^3\left(\sqrt{g}\right)^3 + \ldots \quad (4)$$

Such the form of field dependence of the refractive index enables one to extract some interesting information about the behaviour of n. In a zero approximation (accounting for only the first term in the series (3) or (4)) we obtain:

$$n_0 = 1, \quad (5)$$

thus implying that the refractive index of the so-called polarized vacuum (in terms of the work [16]) is equal to unity. The first approximation (accounting for the next term) gives the relation

$$n_1 = 1 + 4\frac{GM}{2rc_0^2} = 1 + \frac{r_{Sch}}{r} = 1 + 4\frac{\sqrt{GM}}{2c_0^2}\sqrt{g}. \quad (6)$$

This is the so-called weak-field approximation, when the refractive index is proportional to $\sqrt{g}$. The further approximation results in

$$n_2 = 1 + 4\frac{GM}{2rc_0^2} + 7\left(\frac{GM}{2rc_0^2}\right)^2 = 1 + 4\frac{\sqrt{GM}}{2c_0^2}\sqrt{g} + 7\left(\frac{\sqrt{GM}}{2c_0^2}\right)^2\left(\sqrt{g}\right)^2, \quad (7)$$

i.e., the refractive index depends not only on $\sqrt{g}$, but on $\left(\sqrt{g}\right)^2$, too.

In order to illustrate the accuracy of the above approximations, we have calculated, according to Eqs. (1) and (5)–(7), the refractive index of the polarized vacuum on the surface of some astronomical objects (see Table 1).

The data collected in Table 1 shows that higher-order approximations are significant when dealing with strong gravitation fields. Let us now analyse Eq. (7). In our recent paper [13] we have shown that the gravitation constant represents the constitutive coefficient of the polarized vacuum. In frame of the parametric optics and after accounting for nonlinear field dependences of the refractive indices (or the optical-frequency impermeability coefficients), representation of the constitutive coefficient as a function of fields is commonly used (see, e.g., [17]). Thus, one can rewrite Eq. (7) in the following form:

$$n = 1 + \left(4\frac{\sqrt{GM}}{2c_0^2} + 7\left(\frac{\sqrt{GM}}{2c_0^2}\right)^2\sqrt{g}\right)\sqrt{g} \quad (8)$$

In this context, Eq. (8) describes also the changes of the coefficient $\sqrt{GM}/2c_0^2$ with the gravitation field:

Table 1. The refractive index values on the surface of some astronomical objects calculated with Eqs. (1) and (5)–(7).

	Sun	White dwarf ($M \approx M_{Sun}$, $r \approx r_{Earth}$)	Neutron star ($M \approx 1.4M_{Sun}$, $r \approx 10$ km)	Black hole
n_0	1.0	1.0	1.0	1.0
n_1	1.00000425	1.000464	1.414	2.0
n_2	1.00000425…$+8\times10^{-12}$	1.000464009	1.489	2.4375
n	1.0000045	1.001857	1.499	2.6042

Table 2. The changes in the gravitation constant under the influence of gravitation field calculated according to Eq. (12) for the surfaces of some astronomical objects.

$G_0 = 6.672\times10^{-11}$ Nm^2/kg^2			
Object	Earth	Sun	Neutron star
ΔG	8.125×10^{-20}	2.478×10^{-16}	2.418×10^{-11}
$\Delta G/G_0$	1.22×10^{-9}	3.7×10^{-5}	0.36

$$4\frac{\sqrt{GM}}{2c_0^2} = 4\frac{\sqrt{G_0 M}}{2c_0^2} + 7\left(\frac{\sqrt{G_0 M}}{2c_0^2}\right)^2 \sqrt{g}\,, \quad (9)$$

where G_0 and G are the gravitation coefficients referred respectively to the cases of absence and presence of the strong gravitation field. Finally, the dependence of the gravitational coefficient on the gravitation field may be written as

$$\sqrt{G} = \sqrt{G_0} + \frac{7}{4}\frac{\sqrt{M}}{2c_0^2} G_0 \sqrt{g}\,, \quad (10)$$

whereas the corresponding increment is as follows:

$$\Delta G = G - G_0 = = \frac{7}{2}\frac{\sqrt{M}}{2c_0^2} G_0^{3/2} \sqrt{g}\left(1 + \frac{7}{8}\frac{\sqrt{M}}{2c_0^2}\sqrt{G_0}\sqrt{g}\right), \quad (11)$$

$$\Delta G \approx \frac{7}{2}\frac{\sqrt{M}}{2c_0^2} G_0^{3/2} \sqrt{g}\,. \quad (12)$$

Thus, as a result of the mentioned mathematical operations, one can conclude that the coefficient G of gravitational interaction is not exactly a constant and it depends on the gravitation field strength. The illustrations of possible G changes on the surface of some astronomical objects are listed in Table 2 (the corresponding values are calculated according to Eq. (12)).

It is seen that notable values of the gravitation field (for example, the one peculiar for the surface of neutron stars) may cause essential changes in the gravitation coefficient. It is interesting to notice that the changes of G within the Solar System distances could, in principle, be a reason for the observable anomaly of the acceleration of Pioneer 10/11 [18].

Conclusions

It has been shown that the approach based on introduction of the “effective refractive index” for description of light propagation near the massive body or the approach of the polarized vacuum allow to incorporate successfully the approved methods of parametric optics, when describing the influence of gravitation field on the propagation of electromagnetic radiation. One of the most interesting and important conclusions following from this approaches is that the fundamental coefficient of gravitational interaction depends itself on the gravitation field strength.

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