Vortex Solitons, Soliton Clusters, and Vortex Lattices

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Abstract

We introduce novel types of self-trapped extended optical structures, which can be generated in both self-focusing and self-defocusing nonlinear media in the form of *two-dimensional vortex lattices*. We discuss a link of these novel objects with other types of spatially localised self-trapped states, such as vortex solitons and ring-shaped rotating clusters of solitons.

Keywords: optical vortex; soliton cluster; self-trapped periodic wave

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Introduction

Wavefront dislocations of light beams are associated with robust defects in the phase distribution of the field. They appear as semi-infinite lines, called edge dislocations, or as point-like defects with a twisted phase, resembling the structure of fluid vortices. Because the phase has a singular value at the dislocation core, the light intensity vanishes and optical beam acquires a complex structure, usually associated with a higher-order mode of an optical beam, or an *optical vortex*. The study of phase discontinuities in optics now emerges as a separate discipline, the *singular optics* [1].

In *nonlinear media*, the singular beams remain localised because of the self-trapping mechanism that compensates beam diffraction, and the embedded phase dislocation determines a complex internal structure of the beam. Furthermore, the nonlinear self-action may result in a stationary propagation of such light beams, with both intensity and phase pattern remaining unchanged along the propagation direction. In this case, the so-called *spatial optical soliton* is formed [2]. Examples of higher-order spatial solitons in isotropic self-focusing nonlinear media include multipole vector solitons [3] with edge phase dislocations and optical vortex solitons [4-6] (see also a recent review [7] for a comprehensive list of references in the topic of *nonlinear singular optics*).

In this paper, we consider a generalization of the concept of spatial vortex solitons to infinitely extended systems, and introduce novel types of two-dimensional nonlinear lattices with a singular phase structure in isotropic nonlinear media, the so-called *vortex lattices*. The interest to nonlinear self-trapped periodic waves [8,9] has recently been renewed in the context of *optically induced photonic lattices* [10]. Nonlinear diffraction-free light patterns in the form of stable self-trapped periodic waves can exist in many types of nonlinear systems, and they provide a simple realization of *nonlinear photonic crystals*. Such structures are flexible because the lattice is modified and shaped by the nonlinear medium; these flexible lattices extend the concept of optically induced photonic gratings beyond the limit of weak material nonlinearity. Moreover, nonlinear lattices offer many novel possibilities for the study of nonlinear effects in periodic systems, because they can interact with localised signal beams through the cross-phase modulation effect and form composite bound states [11]. In this context, the vortex lattices hold a potential to provide additional means to control and manipulate a signal beam through the mutual guiding and exchange of field momenta.

In this paper, we first present a brief overview of two fundamental objects, vortex solitons and soliton clusters. We argue that a natural generalization of these two concepts to the case of infinitely extended states or nonlinear lattices is a structure created by periodically repeated arrays of soliton clusters extended into two directions. The nonlinear lattices obtained in this way should be associated with a nontrivial phase pattern in the form of a lattice of vortices. We demonstrate that such kind of self-localised nonlinear states exist, as periodic solutions of nonlinear wave equation, in both self-focusing and self-defocusing nonlinear media.

Vortex solitons

We consider propagation of a paraxial laser beam in an isotropic bulk nonlinear medium. Since two-dimensional beams in a Kerr nonlinear medium are known to undergo a catastrophic self-focusing (collapse) instability, we select a model with saturation of the refractive index n,

$$n = n_0 + \varepsilon\, n_2 \frac{I}{I_{sat} + I}, \qquad (1)$$

where n_0 is the linear refractive index of the dielectric medium, n_2 the nonlinear coefficient, I_{sat} the characteristic saturation intensity of the medium, and I the intensity of the propagating light beam. Examples of saturable nonlinear media include resonant rubidium vapours and photorefractive crystals under applied biasing electrostatic filed. In the latter case, the nonlinear coefficient is proportional to the biasing filed and it changes its sign for a reverse polarity of the bias. We reflect this by introducing a coefficient ε, so that the nonlinearity is of a self-focusing type if $\varepsilon = +1$, and it is self-defocusing for $\varepsilon = -1$.

Propagation of paraxial beams with the main wave vector in the direction z and slowly varying complex envelope $E(x,y;z)$ is governed by a quasi-optical equation usually referred to as the nonlinear Schrödinger equation (NLSE), which we write here in the dimensionless form:

$$i\frac{\partial E}{\partial z} + \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \varepsilon F\left(|E|^2\right)E = 0\,, \qquad (2)$$

where the potential F is given by a nonlinear part of the refractive index (1) and, after normalization, $F\left(|E|^2\right) = |E|^2/\left(1+|E|^2\right)$, so that the intensity $|E|^2$ is measured now in units of saturation intensity I_{sat}.

Spatial solitons are the localised optical beams, whose intensity does not change with propagation; such stationary "nonlinear eigenmodes'' of Eq. (2) can be found in a generic form, $E(x,y;z) = U(x,y)\exp(ikz)$, with $U(x,y)$ being the envelope and k the propagation constant. The latter indicates the correction to the wave vector introduced by nonlinearity; this correction depends on the light intensity. Therefore, the family of stationary solutions (solitons) is parameterised by k.

Optical vortices have been introduced as the first example of stationary light beams with a phase twisted around the centre [4], and their structure can be represented as $U(x,y) = R(r)\exp(im\varphi)$. The field should change periodically around the beam core, so the phase twist should be proportional to 2π, with an integer coefficient m; the latter is usually regarded as a topological charge of the phase dislocation. Because the phase in undetermined at the vortex centre, its intensity vanishes and the beam possesses a characteristic shape of a "doughnut mode" (see Fig. 1). Later, it has been shown that higher-order spatial solitons are subject of symmetry-breaking instability, resulting in splitting of the initial doughnut-mode structure into a number of fundamental solitons. The number of splinters and their dynamics are determined by the topological charge of the phase dislocation and the corresponding angular momentum [5] (see [6,7] for an overview on optical vortex solitons).

Soliton clusters

An important step towards understanding of higher-order self-trapped nonlinear beams has been made in Refs. [8,9], that introduced *azimuthally modulated* nonlinear localised structures, the so-called "necklace" beams. It has been shown that a combination of the edge-type phase dislocations with π-out-of-phase neighbouring peaks cannot produce stationary but only slowly expanding "necklaces". Such stabilization is indeed possible for other cases including the attractive interaction between several incoherent beams [10]. Another approach to this problem is a combination of screw dislocation in the origin of a ring-shaped beam with edge dislocations within the necklace [12,13]. The screw dislocation introduces a centrifugal force to the ring that is also responsible for spiraling and mutual repulsion of the splinters in the case of vortex break-up [5], and the edge dislocations prevent noise-induced instability of the ring. Because of nonzero angular momentum in this case, the whole structure rotates while the beam is propagating. As a result, stabilization of ring-shaped multi-hump beams requires a complex phase characterised by a *fractional spin* [13,14].

The phase distribution necessary for the formation of quasi-stationary higher-order self-trapped optical beams has been suggested in Ref. [15] by introducing the concept of *soliton cluster*. In frame of this approach, the azimuthally modulated beam is regarded as *a bound state* of interacting fundamental solitons. Because of the phase-sensitive interaction, the requirement of balance of the interaction forces between the neighbouring solitons determines the beam phase in the form of a staircase-like screw dislocation. Fig. 1 allows one to compare the vortex phase dislocation (left) with the phase of a four-soliton cluster, having well-defined π/2-steps or edge dislocations between the solitons (right). It has been found [15] that the *radially stable* dynamic bound state is formed if these phase jumps, say θ, satisfy the condition $\theta = 2\pi\ m/N$, with $N \geq 4m$ being the number of solitons in the cluster.

Stability of the soliton clusters has been studied numerically in different nonlinear

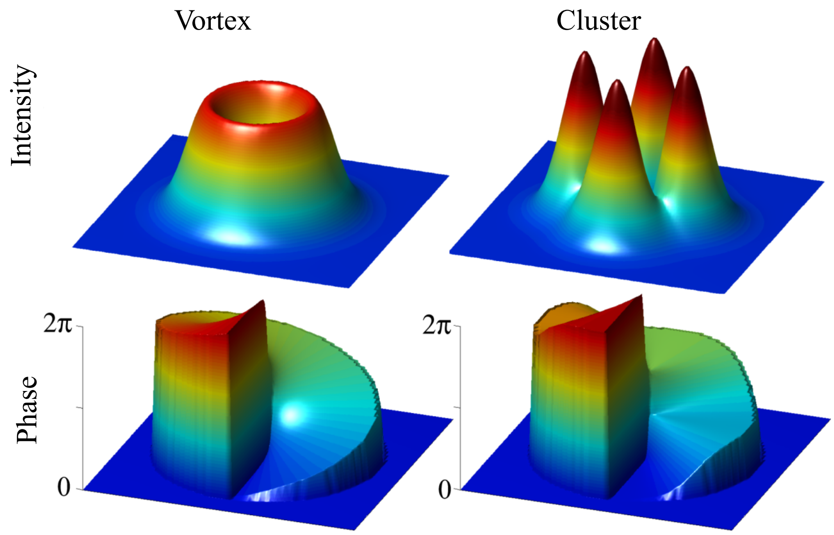

Fig. 1. Structure of the beam intensity and phase for an optical vortex soliton (left) and a cluster of four fundamental solitons (right). In terms of azimuthal coordinate $\varphi = \tan^{-1} y/x$, the vortex phase is given as a *linear function*, $m\varphi$, with integer *m*, while the staircase-like cluster phase is a *nonlinear function*.

media, including the cases of cubic saturable, competing cubic self-focusing and quintic self-defocusing, and competing quadratic and cubic self-defocusing nonlinearities [15,16]. This concept has been extended to higher dimensions, namely to the case of spatiotemporal solitons or light bullets [17]. The common outcome of these investigations is a numerically observed robustness of the soliton clusters to random noise and strong radial perturbations. In the latter case, the pulsating states viewed as radial excitations of the soliton molecule have been observed. Nevertheless, similar to the vortex solitons, the soliton clusters suffer from the symmetry-breaking modulational instability, and they are unstable with respect to azimuthal perturbations. The remarkable feature of this instability is that the number of splinters (the fundamental solitons flying off the ring) is determined mainly by the topological charge m instead of the initial soliton number N, similar to the vortex solitons [5,7].

Vortex lattices: periodic self-trapped nonlinear singular waves

Nonlinear photonic structures created by two-dimensional lattices of pixel-like solitons have recently been demonstrated experimentally in parametric processes [18] and in photorefractive crystals with both coherent [19] and partially incoherent [20] light. For the case of two-dimensional lattices of in-phase solitons created by the amplitude modulation, every pixel of the lattice induces a waveguide, which can be manipulated by an external steering beam [19,21]. However, the spatial periodicity of these lattices is limited by the attractive soliton interaction that may lead to their strong instability. In contrast, the recently suggested two-dimensional lattices of out-of-phase solitons are known to be robust in isotropic saturable model [8], and have been also generated experimentally in anisotropic photorefractive crystal [9]. The phase profile of such self-trapped waves resembles chessboard with the

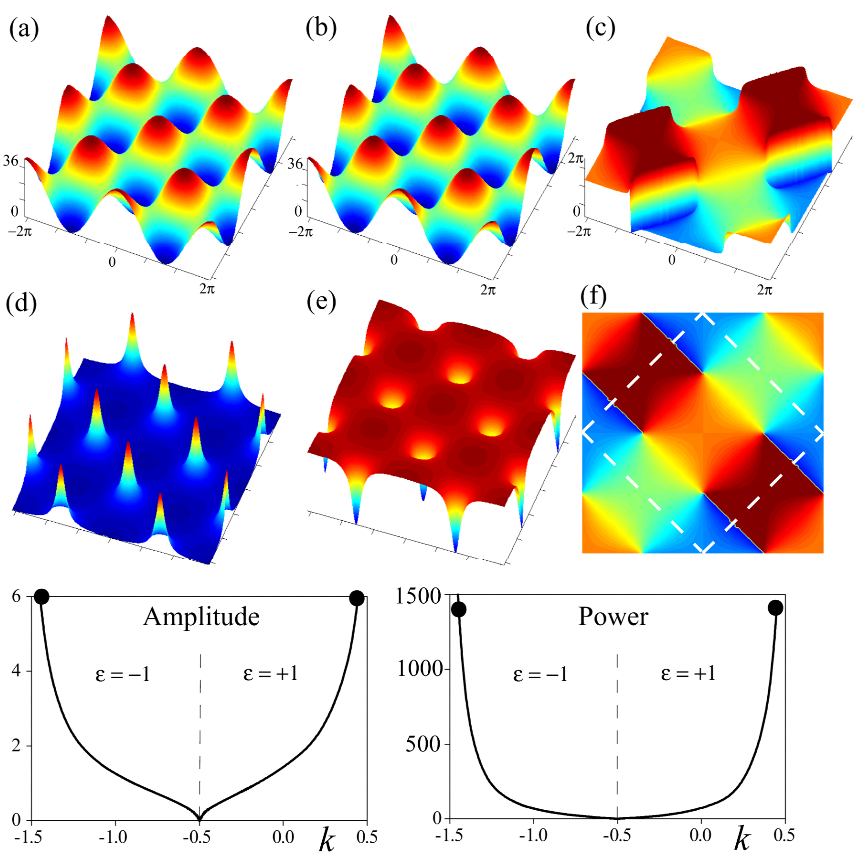

Fig. 2. Self-trapped periodic lattices of vortices in a saturable nonlinear medium. (a,b) – intensity distribution for the periodic solutions in high-saturation regime (the peak intensity >36) for the self-defocusing ($\varepsilon = -1$, $k = -1.45$) and self-focusing ($\varepsilon = +1$, $k = 0.45$) media, respectively (notice that the difference between two plots is hardly visible). (d,e) – corresponding nonlinear refractive index corrections are mutually "inverted". The modes have practically identical phase profiles shown in (c); the top view is shown in (f). Families of solutions are summarised in the diagrams for the amplitude and power of the elementary cell vs. the propagation constant k, and the two dots correspond to the cases (a,d) and (b,e). The elementary cell contains four vortices and is indicated in (f) by a dashed square.

lines of π-phase jumps between neighbouring white and black sites.

Here we expand the concept of the periodic self-trapped nonlinear modes and study *complex* field patterns, carrying phase dislocations. In linear media, such diffraction-free waves are well known. They can be constructed as a special superposition of two real modes, similar to the structure of optical vortex with the phase factor $\exp(im\varphi)$. This can be presented as a superposition of two Laguerre-Gaussian modes, $[\cos(m\varphi) + i\sin(m\varphi)]$. First, we identify two-dimensional stationary periodic solutions of Eq. (2) without the potential, i.e. at $F = 0$. We restrict our study to a square geometry, so that the linear diffraction-free mode is given by

$$E_{lin}(x,y;z) = A\cos(xd)\times \\ \times\cos(yd)\exp(-2id^2z), \tag{3}$$

with the period of the intensity distribution $T=\pi/d$ in both directions x and y and arbitrary amplitude A. From this solution, there appears a bifurcation into the nonlinear regime; it has been studied [8] for an isotropic model with saturation and for a model with anisotropic photorefractive nonlinearity [9]. In the latter case, the optically induced refractive index depends on the orientation of the lattice with respect to the crystal axis. Nonetheless, the nonlinear wave has been found to be robust to such anisotropic deformations and this is experimentally observed [9].

A linear singular periodic wave could be constructed as the superposition

$$E_{lin}^{vortex}(x,y;z) = \\ A\{\cos(xd)\cos(yd) + i\sin(xd)\sin(yd)\}\times, \\ \times\exp(-2id^2z) \tag{4}$$

because the mode (3) is degenerate with respect to rotations and phase shift. Looking for the stationary solutions to the nonlinear model (2), we employ a relaxation numerical procedure (similar to that used in [9]) with an initial trial function being the linear solution given by Eq. (4). We reveal the existence of self-trapped vortex lattices in saturable media with both self-defocusing ($\varepsilon = -1$) and self-focusing ($\varepsilon = +1$) nonlinearities. The examples of these solutions and the soliton-family characteristics are shown in Fig. 2.

Here we stress several properties of these lattices. First, it is well known that the optical vortices in self-defocusing media can only exist on an infinite background and form dark spots; they are usually referred to as *dark vortex solitons* [6]. In contrast, in self-focusing media, the vortex is spatially localised in the form of a bright ring, or a *bright vortex soliton* [7]. Remarkably, this difference is *not important* for the periodic lattices of vortices with the same symmetry, as is seen from Fig. 2. Moreover, we find that the intensity and phase of the lattices are very close to the linear solution (4), with the main difference that the amplitude A is now dependent upon the wave vector k.

Furthermore, we examine the phase structure shown in Fig. 2 (c and f) in more details and find that it reflects the main features of the soliton cluster (cf. Fig. 1, right). Indeed, instead of a linear phase growth around the core of each dislocation, as one would expect for a single vortex soliton, the phase exhibits jumps between the intensity maxima. These jumps are analogous to the (soft) edge-type dislocations, and they define and support a multi-humped structure of the soliton cluster [15,16]. Really, the periodic structure with a square geometry in Fig. 2 involves a four-fold symmetry of the phase modulation, similar to the four-soliton cluster in Fig. 1. At the same time, single-charged clusters can be constructed with any number of solitons ≥4, e.g., five solitons [15], while there is no five-fold symmetry in the periodic structures. Similar symmetry considerations show that the existence of the lattices with higher dislocations, e.g., with double-charged vortices or with a combination of dislocations of different orders (super-lattices), is a highly nontrivial issue. Moreover, the stability properties of these structures should be significantly different in (de-) focusing nonlinear media.

Finally, we note that self-trapped nonlinear waves propagate in the potential (the refractive index profile) they induce, and they are eigenmodes of this potential. We plot the optically induced refractive index distribution in Fig. 2 (d and e) for both types of the modes with $\varepsilon = \pm 1$ and observe that, despite of the fact that the two potentials are mutually inverted, the intensity profiles of their eigenmodes are practically identical. We choose the high-saturation regime to stress a difference between the two potentials. These plots also show a distinction of photonic structures optically-induced by vortex lattices. Namely, fine features of the refractive index landscape, such as the spikes in Fig. 2d, are determined by the vortex core size and, therefore, can be small. One may search for the analogues with, e.g., an array of nanotubes, presented in Ref. [22] as "antennas for visible light".

Conclusions

We have predicted the novel types of two-dimensional optically induced nonlinear photonic lattices associated with a nontrivial phase pattern of self-localised optical beams in nonlinear media. Such vortex lattices provide a nontrivial generalization of the concept of spatial optical solitons. In the self-focusing nonlinear media, they can be described as an infinite lattice of soliton clusters.

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