Singular Beam Diffraction by the Edge of a Dielectric Medium

Izdebskaya Ya.V., Shvedov V.G., Volyar A.V.

Taurida National V.Vernandsky University, 4 Vernadsky Ave., 95007 Simferopol, Crimea, Ukraine e-mail: volyar@ccssu.crimea.ua

Received 09.09.2004

Abstract

Diffraction of Gaussian beam by the system of successive optical wedges is considered. It is shown that the system can form high-order optical vortices. The effectiveness of the system is about 90%. The number of wedges in the system defines the topological charge value.

Key words: optical wedge, singular beam, high-order optical vortices

PACS: 42.25.Fx

Creation of high-order optical vortices has been hitherto based on the phase-transparency technique [1], the computer-generated holograms [2] and the polarized transformations in uniaxial crystals [3]. It has been considered for a long time that the high-order optical vortices are structurally instable due to the light diffraction by an aperture. Recently, we have shown [4] that a non-singular laser beam can be converted into a chain of optical vortices by the dielectric wedge. The obtained beams have specific properties, which are absolutely different from those in the Laguerre-Gaussian beams. Their main property is structural stability.

The aim of the present paper is to study physical mechanism for high-order optical vortices produced by the dielectric-wedge system.

Let the fundamental Gaussian beam with the wave function

$$\Psi_0(r,z)=\frac{\exp(-ikz)}{\sigma}\exp\left(-\frac{r^2}{\rho^2\sigma}\right) \qquad (1)$$

($r^2=x^2+y^2$, $z_0=k\rho^2/2$ being the Rayleigh length, $\sigma=1+iz/z_0$ and ρ the waist radius at z=0) be diffracted by the edge of the dielectric wedge, which has the refractive index

$$n(x)=\begin{cases} n_w, & x>0 \\ 1, & x\le 0 \end{cases}, \qquad (2)$$

where n_w denotes the refractive index of the transparency.

The wave function of the beam at the wedge plane z=0 is

$$\begin{aligned}\Psi_w(x,.y,0)&=\Psi_{00}(x,y,0)\times\\ &\times\exp\{iky[n(x)-1]\times tg\alpha+ikd\}\end{aligned}, \qquad (3)$$

where d is the wedge thickness at the beam axis:

$$d=[n(x)-1][2m+1]\frac{\lambda}{2},\; m=0,1,2,3\,\ldots. \qquad (4)$$

As shown in the work [4], the wave function of the beam in the far-field diffraction region has the form:

$$\begin{aligned}\Psi_1(x,y,z)&\sim\frac{i\pi}{2k}\frac{z}{\sigma}\Psi_0(x,y,z)\{\exp(iky\tan\beta)\times\\ &\times\exp(i\theta)erfc\left[\frac{\sqrt{ik}\;x}{\sqrt{2z\sigma}}\right]+erfc\left[-\frac{\sqrt{ik}\;x}{\sqrt{2z\sigma}}\right]\}\end{aligned} \qquad (5)$$

where $\theta=kd$ is the additional phase of the wave transmitting through the wedge substrate.

The diffraction process presented by Eq. (5) enables us to transform the smooth Gaussian beam into the singular beam, using the chain of optical vortices. In order to select the single optical vortex from the vortex chain, it is necessary to meet the phase matching conditions:

$$\left(\sqrt{\frac{2}{\pi}}+X\right)\exp\left(-X^2\right)=\sqrt{\frac{2}{\pi}} \qquad (6)$$

where $X=\sqrt{kz_0}A$ and $A=\left(n_w-1\right)tg\alpha$, so that the thickness of the wedge substrate satisfies the requirement:

$$h=m\lambda\,,\ m=1,2,3,\dots\,. \qquad (7)$$

Now let the beam transmit through two successive wedges (see Fig. 1). The wedges have the same slope angles β_1 and β_2, while the transparencies are inclined at the angle α to each other. The beam transformed by the system of optical wedges may be presented as

$$\Psi_2(x,y,z)=\frac{1}{kz}\iint_{S^/}dS^/\Psi_1(x^/,y^/,0)\times$$
$$\times\exp\left\{-i\frac{k}{2}\frac{\left(x-x'\right)^2+\left(y-y'\right)^2}{z}\right\}\times \qquad (8)$$
$$\times\exp\{ik\left(d_2\,n(x_1')-z\right)\}\times$$
$$\times\exp\{iky'n(x_1')\tan(\beta_2)\}$$

where d_1 and d_2 are the thicknesses of the first and second wedges at the beam axis and $(x_1^/,y_1^/=0,z=0)$ stands for the boundary of the second wedge, so that we have

$$\begin{cases}x^/{}_1=x^/\cos(\alpha)-y^/\sin(\alpha)\\ y^/{}_1=y^/\cos(\alpha)+x^/\sin(\alpha)\end{cases}. \qquad (9)$$

The presence of the second wedge changes the phase matching condition (7). The thickness d_1 of each wedge at the beam axis has to meet the following requirement:

$$d_1=\frac{\lambda}{2}\frac{(2m+1)}{(n_m-1)},\qquad m=0,1,2\dots\,. \qquad (10)$$

Then the phase difference between the part of the beam transmitting through the wedge and the beam spreading in air would be

$$\theta=\pi\,(2m+1),\ m=0,1,2\dots\,. \qquad (11)$$

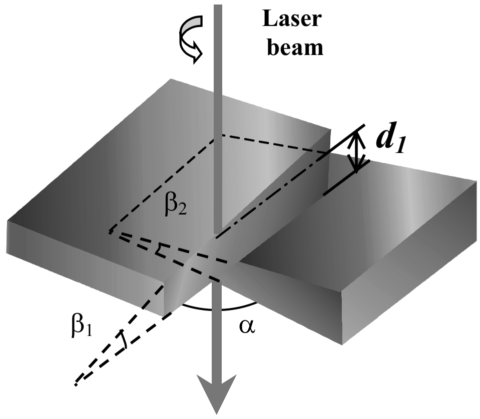

Fig. 1. The sketch of the double-wedge system.

Consequently, the thickness d_2 of the second wedge is

$$d_2=(m+1)d_1,\qquad m=0,1,\dots\,. \qquad (12)$$

Let us rotate the wedge around the Gaussian beam axis by α (Fig. 1). Consider the two cases of mutual orientation of the wedges:

(1) The wedges are positioned so that α=0 and $\beta_1=-\beta_2$.

If $0\le\alpha\le\pi$, the vortex is not formed at the beam axis since the system is equivalent to a plane-parallel plate.

(2) The wedges are positioned so that α=0 and $\beta_1=\beta_2$ (Fig. 2a).

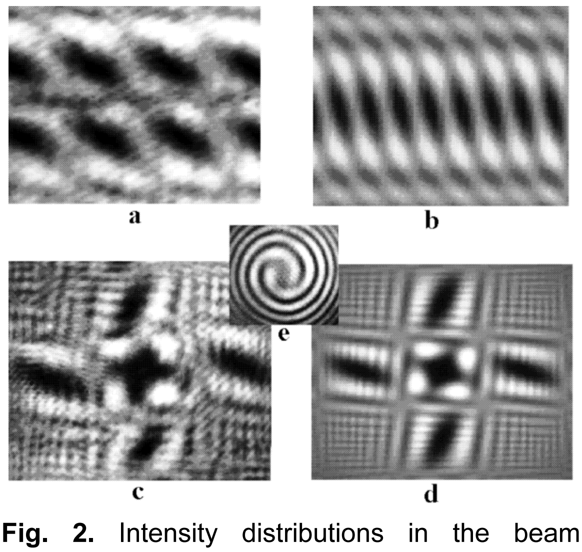

Fig. 2. Intensity distributions in the beam diffracted by the double-wedge system: the vortex chains in the mismatched (a) and matched (b) systems (α=0, π; ρ=3mm; n_w=1.5); experimental (c) and theoretical (d) patterns in the matched system for α=π/2; (e) experimental interference spiral.

This case is of the most interest. Let us consider the situation in a detail:

(a) $\alpha = \pi m$. Then the wave function of the beam after the system of wedges may be expressed, up to a constant factor, as

$$\Psi(r,z) \sim 2\Psi_0(r,z)\left\{ i\,\mathrm{erf}\left(\frac{ikx}{\sqrt{ikz}}\right)\times \right.$$
$$\left. \times\sin[ky(n-1)tg(\beta)] - \cos[ky(n-1)tg(\beta)]\right\} \qquad (13)$$

where n is the refractive index of the wedges.

It follows from Eq. (13) that the number of optical vortices at the boundary between the wedges is doubled but the optical vortex is not generated at the optical axis (see Fig. 2b).

(b) 0<α<π. The topological charge of the optical vortex is now doubled at the beam axis (see Fig. 1c-e). This may be shown with Eq. (8) rewritten in the form

$$\Psi(r,z) \sim \Psi_0(r,z)\left\{ \mathrm{erfc}\left[-\frac{\sqrt{ik}\,x}{\sqrt{2z\sigma}}\right]\mathrm{erfc}\left[\frac{\sqrt{ik}\,y_1}{\sqrt{2z\sigma}}\right] - \exp(ikx\tan\beta)\,\mathrm{erfc}\left[-\frac{\sqrt{ik}\,x}{\sqrt{2z\sigma}}\right]\mathrm{erfc}\left[-\frac{\sqrt{ik}\,y_1}{\sqrt{2z\sigma}}\right] + \right.$$
$$\left. + \exp(ik(x+y_1)\tan\beta)\,\mathrm{erfc}\left[\frac{\sqrt{ik}\,x}{\sqrt{2z\sigma}}\right]\mathrm{erfc}\left[-\frac{\sqrt{ik}\,y_1}{\sqrt{2z\sigma}}\right] - \exp(iky_1\tan\beta)\,\mathrm{erfc}\left[\frac{\sqrt{ik}\,x}{\sqrt{2z\sigma}}\right]\mathrm{erfc}\left[\frac{\sqrt{ik}\,y_1}{\sqrt{2z\sigma}}\right]\right\} \qquad (14)$$

Since we have

$$\lim_{r\to 0}\Psi(r,z) \approx \frac{f(x,y)}{\sigma}\Psi_0(r,z)\exp(i2\phi) \qquad (15)$$

(with $f(x,y,\alpha,\beta_1,\beta_2)$ being responsible for the amplitude distribution in the beam), the double optical vortex is indeed positioned at the beam axis.

Our problem may be extended to the general case of n wedges. We shall in detail consider this case in the experiment.

Experiment

It has turned out that the cover glasses for a microscope have a slight slope of their planes and so could be used as dielectric wedges.

We have selected the two glasses with the identical optical thicknesses and the angles β_1 and β_2 very close to each other. One of the wedges is rotated by the angle α (see Fig. 1) and the intensity distribution in the diffracted beam is observed in the far-field region (i.e., at the distance about 1 m from the wedge). The results obtained are shown in Fig. 2. At first, the edges of the transparencies are slightly moved aside and their verges are parallel to each other (α=0). The system is illuminated with a broad beam (the waist radius is about 3 mm). The diffracted beam forms two vortex chains (Fig. 1a). After the transparencies are moved together, the two chains flow together into a single vortex chain (Fig.2b). The cores of the single vortices in the chain have an elliptic form. The major ellipse axes are equally directed. Besides, all the vortices in the chain have the same topological charge. The rotation of one of the wedges results in transforming the topological structure of the central vortices. Its charge is doubled. At the same time, the topological structure for the rest of vortices remains unchanged (see Fig. 2c,d).

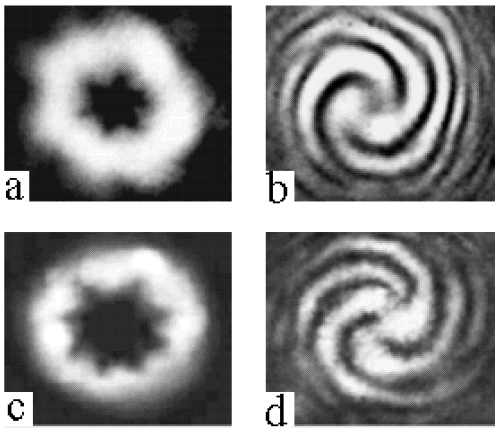

Fig. 3. Experimental intensity distribution (a,c) and interference patterns (b,d) of the optical vortices with topological charges *l*=3 (a, b) and *l*=4 (c, d). The waist of the beam is ρ=1.5 mm.

Similarly, one can construct a system for generating high-order vortices.

The singular beam with the topological charge l=3 is obtained in case of a triple-wedge system provided that $\alpha_1=\alpha_2=\alpha_3=\pi/3$ (see Fig. 3a,b).

The system containing four wedges with the angles $\alpha_1=\alpha_2=\alpha_3=\pi/4$ generates the singular beam with the topological charge l=4 (Fig. 3c,d).

Conclusions

Thus, we have shown both experimentally and theoretically that the system consisting of some optical wedges can generate high-order optical vortices. The energy effectiveness of the system is about 90%. The topological charge of the vortex is defined by the number of wedges in the system.

References

1. Berry M.V. J Opt A: Pure Appl. Opt. 6 (2004) 259-268.
2. Basistiy I.V., Bazhenov V. Yu., Soskin M.S., Vasnetsov M.V. Opt. Comm. 103 (1993) 422-428.
3. Volyar A.V. Fadeyeva T.A. Opt Spectr. 94 (2003) 260-270. (In Russian).
4. Izdebskaya Y.V., Shvedov V.G., Kurabtzev D., Alexeyev A.N., Volyar A.V. Proc. SPIE 4607 (2001) 78-83.
5. Volyar A.V. Fadeyeva T.A. Opt. Spectr. 93 (2002) 286-292.(In Russian).