Optimization of Experimental Conditions for Ellipsometric Studies of ultra-thin Absorptive Films

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Abstract

The method for optimization of experimental conditions necessary for increasing the sensitivity and accuracy of ellipsometric measurements is presented. The method enables to organize an efficient searching algorithm for the optimum conditions. The substantiation of reliability of the method is accomplished. The correlation of results obtained using the present method and the other independent methods is found. The basic attention is devoted to a choice of refractive index of the immersion medium and the angle of incidence of light, in order to ensure the most exact determination for the parameters of absorptive surface films. The technique for the choice of optimum experimental conditions is analyzed for the particular surface models. The areas of global minimum in the experimental error dependences are revealed. The existence of these areas is confirmed with numerical experiments.

Keywords: ellipsometry, absorptive film, surface, inverse ellipsometric problem, optimization, accuracy, sensitivity, errors

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1. Introduction. Geometric interpretation of solution for the inverse ellipsometric problem

It is known that the search of optimum conditions for ellipsometric (EM) experiments in case of arbitrary surface model represents a very complicated problem that has not been fully solved up to date [1-5]. Determination of the parameters of absorptive film (the refractive index n_1, extinction coefficient k_1 and the thickness d_1) for a single-layer surface model is one of the classical problems of ellipsometry [6,7]. The calculation procedures elaborated for this problem may be generalized for the more complicated models. Recently we have elaborated the approximate calculation procedure for the accuracy estimation relevant for absorptive films studied using the null-EM technique [8]. The rapid processing of the data set, which determines the experimental conditions, is essential advantage of this procedure. The principal goal of this article is to check its reliability with the aid of independent methods.

As explained in a number of studies [8-10], the solution of the inverse EM problem may be found for each EM measurement and each pair of experimental values Δ and Ψ. This solution may be represented as some complex spatial curve of a spiral shape in a three-dimensional space of film parameters (n_1,k_1,d_1). We refer to this spiral as a "(ρ=*const*)-contour", using the general expression of ellipsometry $\rho \equiv R_s/R_p = \tan\Psi e^{i\Delta}$ and the fact of constancy of (Δ,Ψ)-parameters for every point of the contour. Here R_s and R_p are the amplitude light reflection coefficients of the surface structure measured respectively for the "s" and "p" incident polarization states. All the curves have a following essential peculiarity: the arbitrary

(d_1=*const*)-plane crosses the spiral contour at a single point only. The true parameters of the absorptive film can be obtained as a result of a series (at least, two) of independent measurements performed under different experimental conditions.

Any change in the experimental conditions (e.g., the angle of incidence φ_0, the refractive index of the immersion medium n_2 or the wavelength λ of incident light) would lead to displacement and changes in the shape of the spiral. Nevertheless, all the contours, which correspond to different experimental conditions and the same surface structure, intersect each other at the single point. Its coordinates (n_1^0, k_1^0, d_1^0) are the true parameters of the film. The example of such intersection is shown in Figure 1 for the two different (ρ=*const*)-contours.

Every mentioned contour corresponds to a certain pair of experimental parameters Δ and Ψ. The experimental errors of determining the EM angles δΔ and δΨ result in displacement of spiral contours in the (n_1,k_1,d_1)-space. The displacement of intersection point of the (ρ=*const*)-contour and the (d=*const*)-plane in the $d_1^0 = const$ plane caused by experimental errors may be obtained from the equation (see [8])

$$\Delta r = \sqrt{\Delta n_1^2 + \Delta k_1^2}\,. \tag{1}$$

Here $\Delta n_1 = \dfrac{\dfrac{\partial\Psi}{\partial k}\delta\Delta - \dfrac{\partial\Delta}{\partial k}\delta\Psi}{\dfrac{\partial\Delta}{\partial n}\dfrac{\partial\Psi}{\partial k} - \dfrac{\partial\Delta}{\partial k}\dfrac{\partial\Psi}{\partial n}}$ and $\Delta k_1 = \dfrac{\dfrac{\partial\Delta}{\partial n}\delta\Psi - \dfrac{\partial\Psi}{\partial n}\delta\Delta}{\dfrac{\partial\Delta}{\partial n}\dfrac{\partial\Psi}{\partial k} - \dfrac{\partial\Delta}{\partial k}\dfrac{\partial\Psi}{\partial n}}$ are the n- and k-components of the displacement that determine the error values for the optical film parameters in the ($d_1 = d_1^0$)-plane (the case of exactly determined film thickness). All the partial derivatives should be taken for the initial surface model (n_1^0, k_1^0, d_1^0).

All the possible displacements associated with the experimental errors form a certain plane rectangle-like figure in the (d= *const*)-plane (see Figure 1). Then the points at the (ρ=*const*)-contour form a spatial figure similar to prism. Two such "prisms" corresponding to different experimental conditions and different pairs of the EM parameters (Δ,Ψ) intersect with each

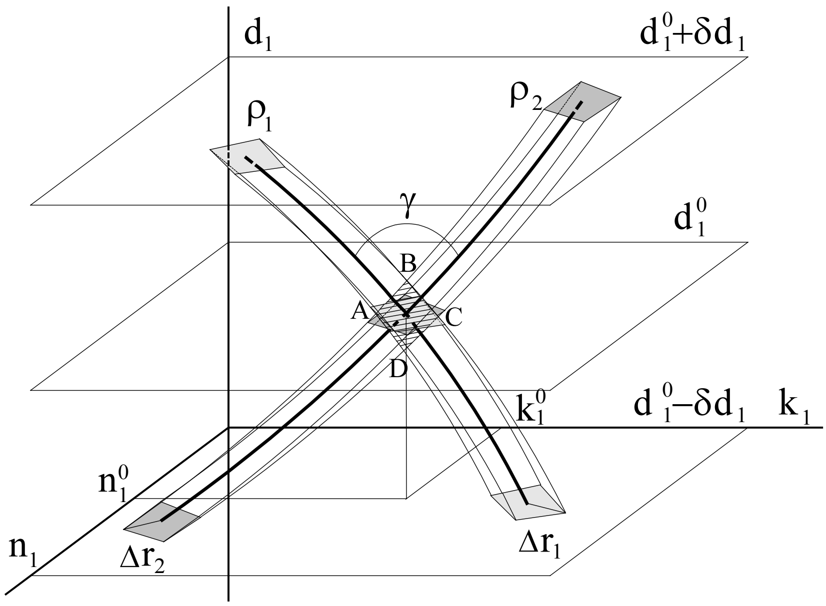

Fig. 1. Intersection of the (ρ=const) spiral contours, which correspond to different experimental conditions; γ is the angle of intersection between the (ρ=const)-contours.

other and produce a complex spatial figure ABCD. Any point inside this figure is a potential solution of the inverse EM problem for the two given sets of experimental conditions. Therefore, the dimensions of this figure determine the error of determining the film parameters.

It is obvious that the dimensions of the said figure depend, first of all, on the values of Δr_1, Δr_2 -displacements and the angle between the (ρ=*const*)-contour tangents at the intersection point. We have earlier suggested [8] the calculation procedure for obtaining the mean-square error for determination of the film parameters:

$$t = \sqrt{(\delta n_1)^2 + (\delta k_1)^2 + (\delta d_1)^2}\,, \qquad (2)$$

where δn_1, δk_1 and δd_1 are respectively the most probable errors for the refractive index, the extinction coefficient and the thickness of the film under test.

In the present work we shall examine closely the given procedure and substantiate its reliability.

2. The accuracy forecast for EM measurements

2.1. Estimation of errors for the optical parameters (n_1, k_1)

In order to solve this problem, we have used the known technique of approximate solution of the inverse EM problem. Namely, we start with the approximate expression for the Drude's equation obtained by Lucy [11]:

$$\frac{\rho}{\rho_0} = 1 + i4\pi \frac{d_1}{\lambda} n_2 \sin\varphi_0 \cdot \tan\varphi_0 \cdot \sigma\,, \qquad (3)$$

where $\sigma = \dfrac{N_1^2 - n_2^2}{N_1^2}\dfrac{N_1^2 - N_0^2}{N_0^2 - n_2^2}\dfrac{N_0^2}{N_0^2 - n_2^2 \tan^2\varphi_0}$,

n_2, N_1, and N_0 are respectively the refractive indices of the immersion medium, the absorptive film and the substrate, d_1 the film thickness, λ the light wavelength, φ_0 the angle of incidence of the light beam, $\rho = \tan\Psi \exp(i\Delta)$ and $\rho_0 = \tan\Psi_0 \exp(i\Delta_0)$, and Δ_0, Ψ_0, Δ, and Ψ the experimental EM angles obtained for the pure substrate surface ($d_1 = 0$) and the "substrate – thin film" structure ($d_1 \neq 0$).

The above approximation holds correct for the film thickness $d_1 \ll \lambda$. For the wavelength in the visible region, the condition $d_1 \leq 5$ nm should be then surely satisfied. The essential advantage of the approximation consists in the possibility for simply reversing equation (3). For the complex refractive index of thin surface film, the following biquadratic equation may be obtained:

$$N_1^4 + BN_1^2 + C = 0\,, \qquad (4)$$

where

$$B = \frac{(N_0^2 - n_2^2)(1 - \dfrac{\rho}{\rho_0})(N_0^2 - n_2^2 \tan^2\varphi_0)}{i4\pi \dfrac{d_1}{\lambda} n_2 \sin\varphi_0 \cdot \tan\varphi_0 \cdot N_0^2} - (n_2^2 + N_0^2)$$

and $C = N_0^2 n_2^2$.

From among the four solutions of equation (4), the two ones with positive real parts should be selected. Furthermore, only one of them would have a real physical meaning.

Such the approximate solution enables one to find a pair of the optical film parameters $N_1 = n_1 - ik_1$, which correspond to arbitrary pair of experimental EM angles Δ and Ψ, for the selected film thickness, surface model and the experimental conditions. In such a way, we may "reflect" any (Δ,Ψ)-region of experimental parameters to the corresponding region of the optical film parameters (n_1, k_1). The analogous (Δ,Ψ) $\rightarrow$ (n_1, k_1) "reflection" may be accomplished using equations (1) for the arbitrary film thickness.

We have carried out comparative calculations for the set of the surface models in case of $d_1 \leq 5$nm, using equations (1) and (4). The regions of the experimental errors $\Delta' \pm \delta\Delta$, $\Psi' \pm \delta\Psi$ (see Figure 2) have been "reflected" to the corresponding (n_1, k_1)-regions on the (d=const) plane. Here Δ' and Ψ' denote the true

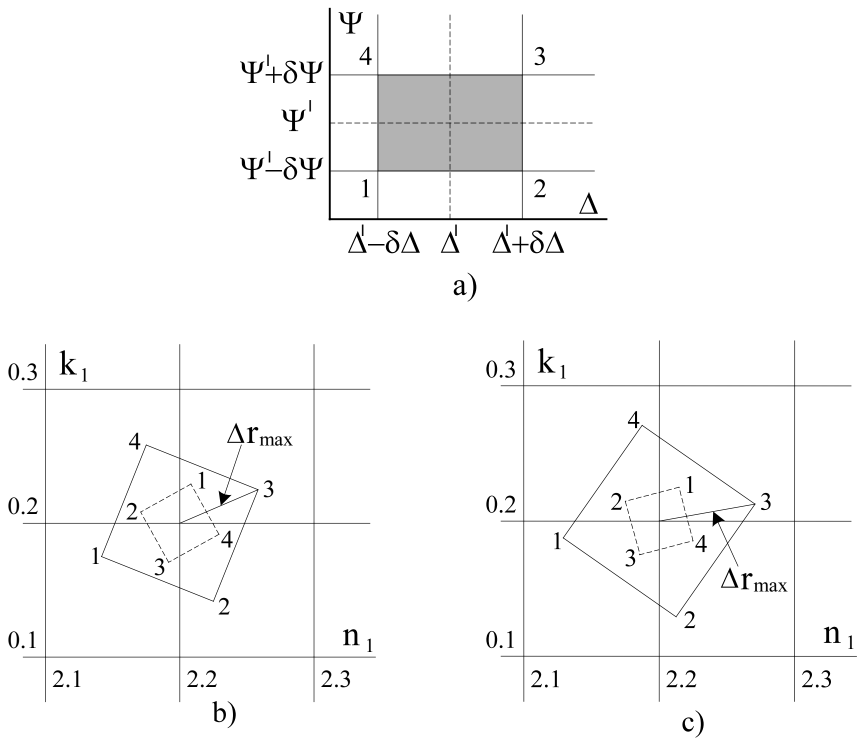

Fig. 2. Region of experimental errors (a) and its "reflections" to (n_1, k_1)-region calculated with (b) equations (4) and (c) equations (1). The dash contours correspond to the angle of incidence φ_0=60 ° and the solid ones to φ_0=40 °.

EM parameters calculated for the selected surface model, and δΔ, δΨ the minimum changes of EM angles, which can be detected with ellipsometer [12]. These values would also depend upon the surface model parameters and the experimental conditions:

$$\delta\Delta = R/(|R_s||R_p|),\ \delta\Psi = 1/R, \qquad (5)$$

where $R = \sqrt{\frac{\left(|R_s|^2 + |R_p|^2\right)}{2}}$.

The results of the numerical experiment are shown in Figure 2. The parameters of the surface model are chosen as follows: $n_2 = 1.0$, $N_1 = 2.2 - i\cdot 0.2$, $N_0 = 1.5 - i\cdot 0$ and $d_1 = 5nm$. The results of $(\Delta,\Psi)\to(n_1, k_1)$ "reflections" are practically identical for both calculation procedures.

As can be easily seen, the maximum errors of the optical parameters (n_1, k_1) caused by the experimental errors δΔ and δΨ are equal to the half-diagonal of the 1234-rectangles. These errors depend essentially on the experimental conditions. Therefore, the comparison of $\Delta r_{\max}(\varphi_0, n_2)$ dependences calculated with equations (1) and (4), represents an informative confirmation of reliability of the suggested calculation algorithm. The dependences $\Delta r_{\max}(\varphi_0)$ calculated with the accurate equation (1) and the approximate one (4) are shown in Figure 3 for the same surface model. Here we assume that the film thickness d_1 is determined without any errors.

We have a satisfactory correlation of the results of calculations in the working range of incidence angles $\varphi_0 \geq 45^{\circ}$. The correlation improves with decrease of the film thickness. The approximate character of equation (3) may explain some difference in the results for the small angles of incidence. The increase in this difference occurring with increasing refractive index n_2 of the immersion medium could be attributed to the (const/0)-peculiarity in the approximate equation (3) at the (n_2 = n_0)-point. The latter takes place only for the surface models with a transparent substrate (k_0 = 0).

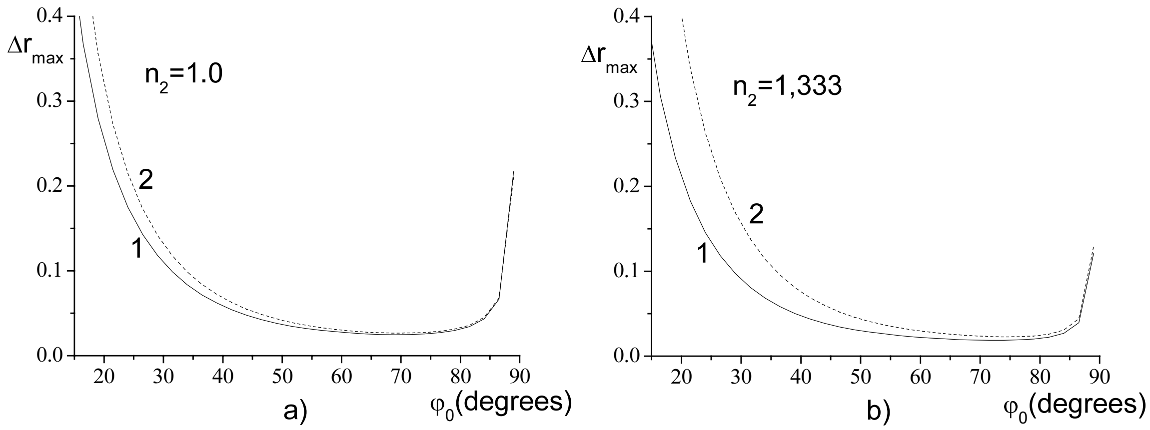

Fig. 3. Dependences of the maximum error of the optical film parameters $\Delta r_{\max}$ on the incidence angle φ_0 calculated using the Lucy's approximation (1) and the suggested procedure (2) for the cases of refractive indices of the immersion medium $n_2 = 1.0$ (a) and $n_2 = 1.333$ (b). $N_1 = 2.2 - i \cdot 0.2$, $N_0 = 1.5 - i \cdot 0$ and $d_1 = 5nm$ are the parameters of the surface model.

2.2. Error estimation for simultaneous determination of the all three film parameters

As mentioned above, all the (ρ=*const*)-contours, corresponding to the same surface model though various experimental conditions, cross at the same point. The coordinates of this point are the real parameters (n_1^0, k_1^0, d_1^0) of the absorptive film. Every contour also intersects an arbitrary (d_1=*const*)-plane $(d_1 \neq d_1^0)$. Therefore, every set of experimental conditions has the corres-ponding point in the arbitrary (d_1=*const*)-plane. The arbitrary region of experimental conditions can be "reflected" this way to the corresponding (n_1,k_1)-region in the $(d_1 \neq d_1^0$=*const*)-plane. The shape and dimensions of this region prove to be very informative. This allows one to estimate the errors of the sought parameters for the every set of experimental conditions.

An example of the (n_1, k_1)-regions calculated for a certain surface model (n_1^0=2.2, k_1^0=0.2, d_1^0=5nm and n_0=1.5, k_0=0) is shown in Figure 4. Here we present the "reflections" of the same rectangular (n_2,φ_0)-region obtained using the two different procedures. The dif-ferences in the dimensions and the displacement of the "reflected" regions are not significant, being caused by the approximate character of equation (3).

It is evident that the variations of refractive index of the immersion medium has a much more notable influence on the displacement of (ρ=*const*)-contours in the (n_1, k_1, d_1)-space (i.e., on the reduction of error of the inverse EM problem) than the incidence angle variations have.

We have used the direct method for checking the suggested procedure, too. One can find the approximate coordinates of the intersection point of the (ρ=*const*)-contours and the ($d_1 \neq$ d_1^0 =*const*)-plane for every set of experimental condition and an arbitrary surface model, using the equations obtained in [8,10]:

$$n_1' = n_1^0 + \Delta d_1 \frac{\frac{\partial \Delta}{\partial k}\frac{\partial \Psi}{\partial d} - \frac{\partial \Psi}{\partial k}\frac{\partial \Delta}{\partial d}}{\frac{\partial \Delta}{\partial n}\frac{\partial \Psi}{\partial k} - \frac{\partial \Delta}{\partial k}\frac{\partial \Psi}{\partial n}},$$

$$k_1' = k_1^0 + \Delta d_1 \frac{\frac{\partial \Psi}{\partial n}\frac{\partial \Delta}{\partial d} - \frac{\partial \Delta}{\partial n}\frac{\partial \Psi}{\partial d}}{\frac{\partial \Delta}{\partial n}\frac{\partial \Psi}{\partial k} - \frac{\partial \Delta}{\partial k}\frac{\partial \Psi}{\partial n}}. \qquad (6)$$

Here $\Delta d_1 = d_1' - d_1^0$ is the distance between the d_1^0=*const* and d_1'=*const* planes. All the partial derivatives should be taken at the initial (n_1^0, k_1^0, d_1^0) point.

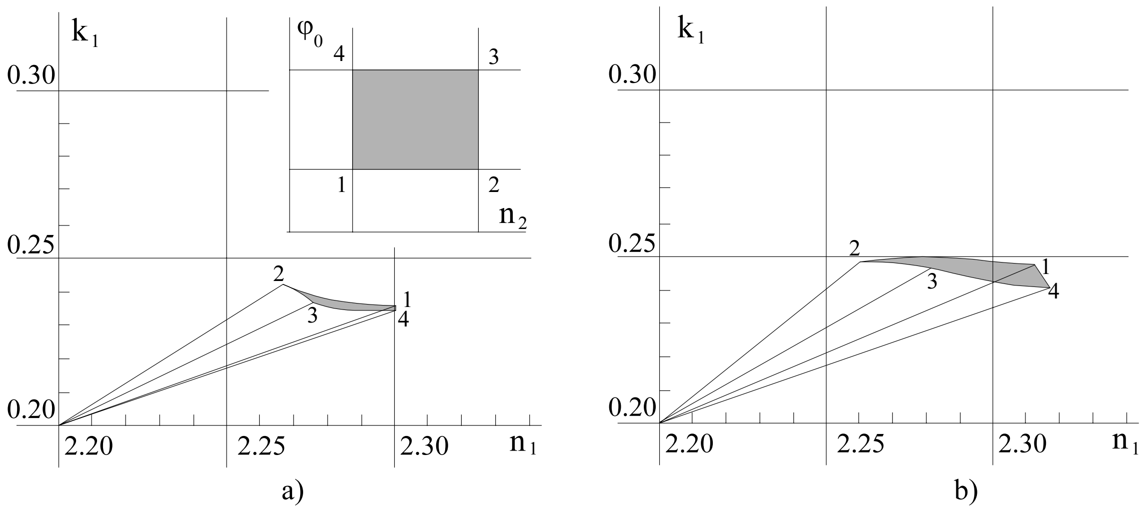

Fig. 4. "Reflections" of (φ_0,n_2)-rectangular region of experimental conditions to the corresponding (d_1=const)-region calculated with the procedure based on equation (1) (a) and the approximate equation (4) (b). The limits of experimental region are 45°<φ_0<90° and 1.0<n_2<1.3 and $d_1 - d_1^0 = -0.5$nm.

The smaller the difference Δd_1, the higher determination accuracy of the intersection point coordinates is. The "new" coordinates n_1', k_1' and d_1' may be used as the parameters of thin absorptive film to be involved into calculations of the EM parameters Δ and Ψ while using the accurate Drude's equation. The difference between the (Δ,Ψ)-parameters of this "new" surface model and our initial model (n_1^0, k_1^0, d_1^0) must have been very small if only the suggested procedure (6) be correct.

The results of numerical experiment are presented in Table 1 for the same surface model (n_0=1.5, k_0=0.0, n_1^0=2.2, k_1^0=0.2, d_1^0=10nm and $\Delta d_1 = d_1 - d_1^0$=-0.5nm). The current parameter, which determines the difference in the (Δ,Ψ)-parameters for the initial and "new" models, is the refractive index of the immersion medium n_2. In this way one can estimate the accuracy of the procedure for different experimental condition. Our calculations have been performed for the angle of incidence φ_0=60° and the wavelength λ=632.8 nm.

If we change the film thickness only ($\Delta d_1 = d_1 - d_1^0$=-0.5nm) within the initial surface model, we have the changes in the (Δ,Ψ)-parameters of about δΔ ≅ 5.0° and δΨ ≅ 0.5°. The obtained values δΔ and δΨ (see Table 1) show therefore a satisfactory accuracy of equations (6). This enables to determine the tangent direction to the (ρ=*const*)-contour at the (n_1^0, k_1^0, d_1^0)-point.

Table 1. Comparison of (Δ,Ψ)-parameters calculated for different surface models with the film parameters belonged to the same (ρ=const)-contour.

n_2	Ellipsometric parameters				δΔ=	δΨ=
	Initial model		New model		=│Δ-Δ′│	=│Ψ-Ψ′│
	Δ(deg)	Ψ(deg)	Δ′(deg)	Ψ′(deg)	(deg)	(deg)
1.0	276.5080	5.6827	276.5900	5.6794	0.0820	0.0033
1.1	302.3929	7.1561	302.4679	7.1575	0.0750	0.0014
1.2	315.0985	9.1164	315.1540	9.1215	0.0555	0.0051
1.3	320.6028	10.4370	320.6419	10.4450	0.0391	0.0080
1.4	324.4043	9.3062	324.4185	9.3162	0.0142	0.0100
1.5	37.7928	6.5374	37.7913	6.5375	0.0015	0.0001
1.6	41.7283	29.3383	41.6961	29.3503	0.0322	0.0120

We may obtain a further interesting confirmation of appropriateness of the suggested procedure. It is obvious (see equation (3)) that the (ρ/ρ_o)-function has the following important peculiarity in the $(n_2 \approx n_0)$-region for the surface models with transparent substrates:

$$\lim_{n_2 \to n_0} \frac{\rho}{\rho_0} = \infty . \quad (7)$$

The (ρ/ρ_o)-ratio characterizes a sensitivity of EM parameters to the film thickness changes. A break off in the (ρ/ρ_o)-function is a consequence of approximations adopted during derivation of equation (3).

The analysis of the accurate Drude's equation shows also an essential increase in sensitivity of EM parameters to (ρ/ρ_o-value) for the case of $d_1 \ll \lambda$ and $n_2 \approx n_0$. It is therefore natural to expect the error value to be reduced in the region $n_2 \approx n_0$ for very thin films and the films satisfying the condition

$$d_1 \approx \lambda/(2n_1\cos\varphi_1) = \lambda/[2(n_1^2 - n_0^2\sin^2\varphi_0)^{1/2}], \quad (8)$$

where φ_1 is the angle of refraction of the light beam in the film material at the condition $k_1=0$.

The procedure based on equation (6) is also suitable for estimating the mean-square error $t = ((\delta n_1)^2+(\delta k_1)^2+(\delta d_1)^2)^{1/2}$. Here δn_1, δk_1 and δd_1 are respectively the errors in determination of the optical film parameters and its thickness. The suggested calculation procedure enables a separate error determination for the film thickness δd_1 and the merely optical parameters $((\delta n_1)^2+(\delta k_1)^2)^{1/2}$. The ratio $A=\delta d_1/((\delta n_1)^2+(\delta k_1)^2)^{1/2}$ depends on the angle between the (ρ=*const*)-contours and the (d_1=*const*)-plane. The steeper the slope of the (ρ=*const*)-contours tangents, the greater the A ratio is. The slope of the (ρ=*const*)-contours decreases with decreasing film thickness, and the contours approach asymptotically the (d_1=0)-plane. This fact represents a general reason for the error increasing in case of optical parameters (δn_1,δk_1) of very thin films.

Nevertheless, we consider the problem of simultaneous determination of all three film parameters. Therefore the minimization of the t-error is a basic task. In case of the selected surface model, the t-error value is a function of experimental conditions. Examples of two-dimensional diagrams of the t-error calculated in the corresponding two-dimensional fields of experimental conditions are shown in Figure 5.

In all the diagrams the angle of incidence is variable parameter for both axes. All the ordinates correspond to the measurements in the air as immersion medium (n_2=1.0) and the

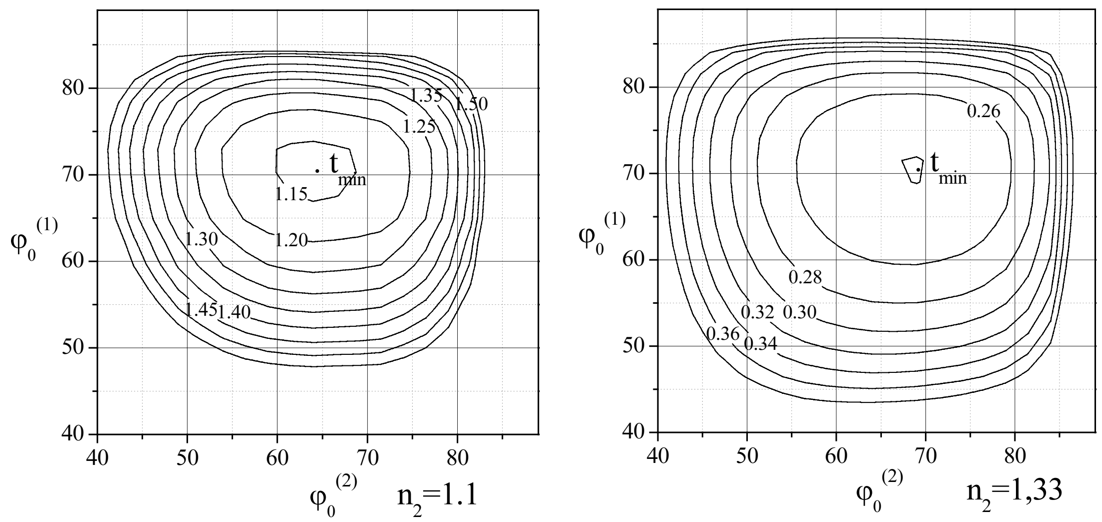

Fig. 5. Two-dimensional diagrams of the mean-square t-error of the film parameters in dependence on experimental conditions for the refractive indices of immersion medium $n_2 = 1.1$ (a) and $n_2 = 1.33$ (b). Here n_1 = 2.2, k_1 = 0.2, d_1 =10 nm, n_0=1.5 and k_0 =0 are the parameters of the surface model.

abscissas to the experimental situations with different immersion media ($n_2 \neq 1.0$). It can be easily seen that the diagrams are very useful for both determining the optimum experimental conditions and minimizing the errors for the film parameters under study. All the diagrams have a pronounced minimum of the mean-square error. This allows one to use successfully the numerical optimization methods for determining the set of optimum experimental conditions. Notice that the displacement and depth of the minimum depend essentially on the refractive index n_2.

The dependences of the minimum t-error versus the immersion refractive index n_2 are especially interesting. They are shown in Figure 6 for different film thickness and surface models. One can see a specific feature of these dependences in the region of $n_2 \approx n_0 = 1.5$ for the film thickness $d_1 < 10$nm and $d_1 = 188 \pm 10$nm.

For the wavelength $\lambda = 632.8$nm and the incident angle $\varphi^0 = 70°$, the film thickness calculated from equation (8) is equal to 188 nm. To satisfy equation (8) in the region $n_2 \approx n_0$, the condition should be fulfilled for the light waves, reflected from the top and bottom film surfaces, to be out of phase by 3π. This is a common condition for the second interference minimum in the reflected light. In the region $\lambda >> d_1 \approx 0$ this phase difference is equal to π (the first interference minimum condition). The efficiency of interference between these waves is the highest for transparent films and decreases for the absorptive ones. As a consequence, the

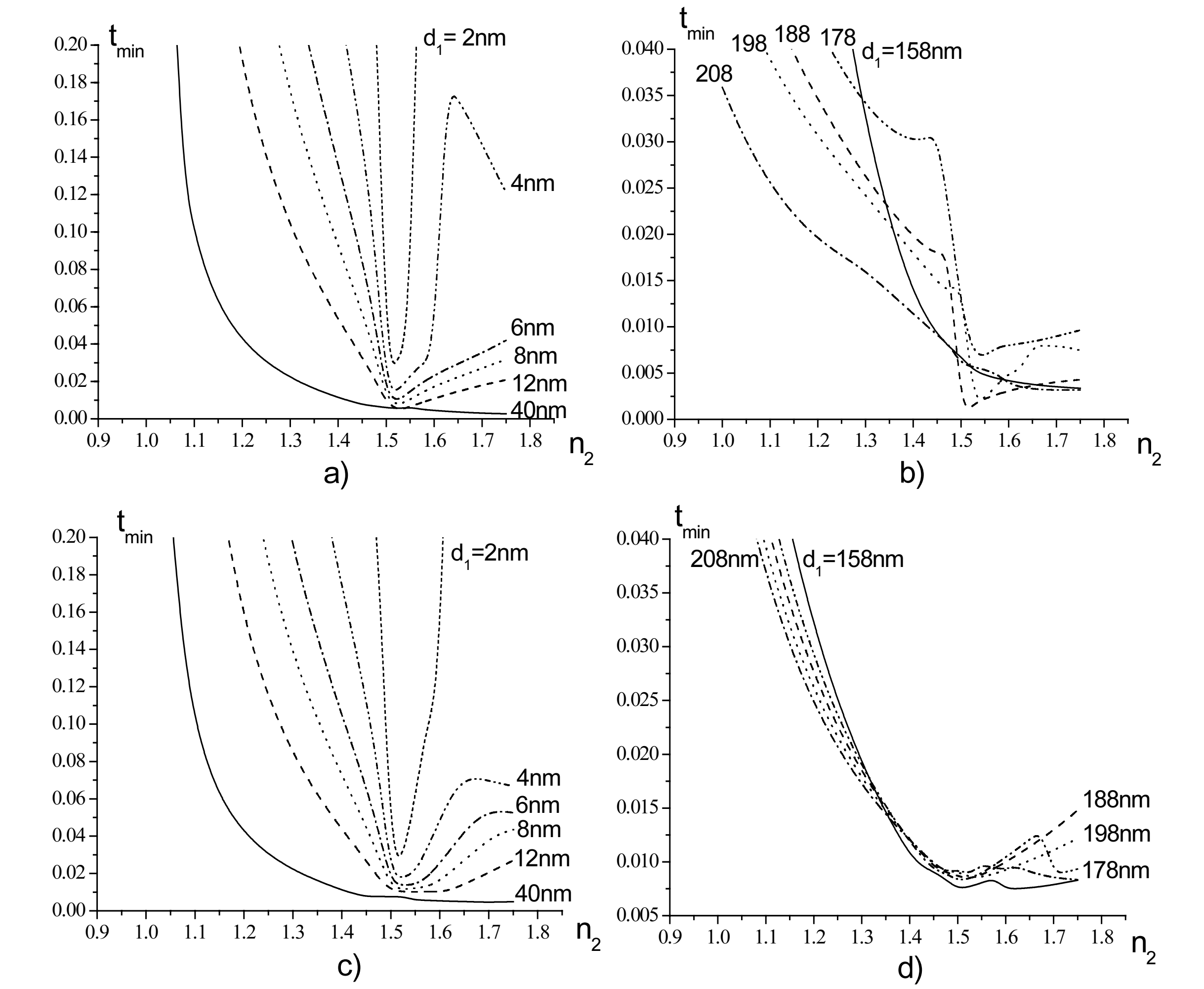

Fig. 6. Dependences of the minimum t-error on the refractive index n_2 of immersion medium for different optical parameters of the surface model and film thickness: n_1 =2.2, k_1=0.05 (a,b), k_1=0.5 (c,d), and n_0=1.5, k_0 =0.

error-reduction effect in the region $n_2 \approx n_0$ must be stronger for more transparent films. Comparison of (a,b) and (c,d) dependences in Figure 6 is a good confirmation of this conclusion. It is obvious that the equality $n_2 = n_0$ is the condition of optical homogeneity of the film surrounding. Very thin films ($d_1 << \lambda$) or the films satisfying the condition (8) represent essential inhomogeneity in the surrounding homogeneous medium and so cause an increase in the intensity of reflected beam. Even insignificant changes in the optical film parameters or its thickness cause considerable changes in the reflected beam intensity and, consequently, the EM parameters change. We therefore expect the error reduction effect for such the experimental conditions and the surface structures. For the absorptive films, the effect must be more vivid in the ($d_1 << \lambda$, $\Delta\varphi = \pi$)-region (i.e., for very thin films) than for the ($\Delta\varphi = 3\pi$)-region.

The results obtained using the suggested calculation procedure confirm completely those conclusions (see Figure 6).

In order to estimate additionally the film parameter errors, a direct computational experiment has been carried out for a set of surface models. We have in particular checked the availability of the error reduction effect in the ($n_2 = n_0$)-region. The EM parameters (Δ_0, Ψ_0) have been calculated for different initial models and the two sets of experimental conditions determined with the aid of the suggested procedure. The latter conditions correspond to the minimum value of the mean-square t-error (see Figure 5). Then a deviation of the EM parameters has been generated for every set of experimental conditions. The deviation values ($\delta\Delta, \delta\Psi$) simulate the experimental errors. They are calculated with equations (5). We use the "new" EM parameters ($\Delta_0 \pm \delta\Delta, \Psi_0 \pm \delta\Psi$) for solving the inverse EM problem and determining the surface film parameters (n_1, k_1, d_1). The deviations ($n_1 - n_1^0$, $k_1 - k_1^0$, $d_1 - d_1^0$) of the "new" parameters from those of the initial model determine the film parameter errors, which are caused by experimental errors ($\delta\Delta$, $\delta\Psi$). These errors may be compared with the t-error obtained using the suggested procedure. We have finally obtained a fairly good correlation of these quantities for different surface models. An example of such the correlation is demonstrated in Figure 7 for a partial case of the surface model. This is another confirmation of reliability of our procedure.

Conclusion

Hence, we have reached our main goal. The calculation procedure suggested by us for estimation of the errors of EM measurements

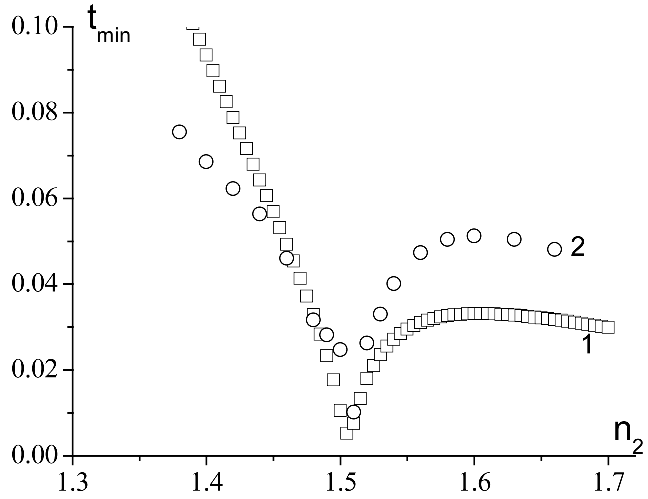

Fig. 7. Comparison of total errors for the film parameters calculated while (1) using the suggested procedure and (2) solving the inverse EM problem.

has been proven for the case of film thickness $d_1 << \lambda$. The results obtained with the Lucy's approximation and those based on our technique are well correlated. The examinations have been performed for a wide range of experimental conditions and a number of surface models, e.g., the model "transparent substrate – absorptive film". We have shown that the suggested method allows one to obtain the tangent direction to the contour with the constant EM parameters ($\rho = const$) at arbitrary point in the film-parameter space (n_1, k_1, d_1). The angle of intersection of any ($\rho = const$)-contours corresponding to various experimental conditions may be found and the errors may be estimated in this manner.

The value of the mean-square error thus obtained may be further used as a target function in the classical problem of minimization. Therefore the searching algorithm for the optimum conditions of EM measurements reduces to minimization algorithm for the function $t = \sqrt{(\delta n_1)^2 + (\delta k_1)^2 + (\delta d_1)^2}$ in a multi-dimensional space of parameters determining the experimental conditions. This enables one to find the optimum conditions of the EM experiment in case when the three film parameters are being determined simultaneously.

Finally, the calculations carried out according to the suggested scheme confirm the predicted sensitivity increase in the EM measurements for the particular cases of surface models and experimental conditions.

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