Methods for the Studies of Piezooptic Effect in Crystals and Analysis of the Experimental Data

II. Analysis of the experimental data

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Abstract

This review article deals with the analysis of works dedicated to studies of absolute piezooptic effect in crystals, which possess a symmetry lower than the cubic one. We demonstrate a possibility to construct indicative piezooptic surfaces, basing on the completed matrices of piezooptic coefficients. The examples for analysis of the piezooptic effect anisotropy are given, which uses the indicative surfaces, as well as their sections and stereographic projections.

Key words: absolute piezooptic coefficients, indicative surfaces, stereographic projections, symmetry classes.

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Content

Introduction

During the last twenty years, the interest to the piezooptic effect (POE) has not been weakening. A major number of works has been concerned with the birefringence induced by mechanical stresses (see, e.g., [1-8]) or the retardation (e.g., [9-15]), including the studies performed in the most recent years [16-20]. As a rule, the majority of these works contain quite objective results, since the polarization-optical method [21,22], which has been used in these cases, ensures a high measurement accuracy (~5%) and a satisfactory reproducibility of the results, without employing researcher's "ruses" and "know-how". This method has been in many ways improved, too [23]. However, we would like to call in question the results of those works, where the authors declare studying the changes in the birefringence induced by uniaxial pressure for the crystals with a large natural birefringence, and, at the same time, they do not take into account the elastic deformation of samples (this problem is perfectly elucidated in [24], chapter 5).

Let us notice that, in particular cases, the birefringence induced by mechanical stresses allows one to calculate the absolute piezooptic coefficients (POCs), which describe the changes in the refraction index or the rotation of optical indicatrix. This concerns to the coefficient $\pi_{44}\equiv\pi_{55}\equiv\pi_{66}$ [22-24] of cubic crystals, the coefficient π_{66} [21,24-29] of tetragonal crystals, which belong to the symmetry classes 422, 4mm, 4/mmm, and $\bar{4}2m$, and also all of 12 absolute POCs in rhombic crystals. The POC matrix of the rhombic crystals does not contain the rotational (or turning), the shift and the off-diagonal turning-shift POCs (the definitions of the turning, shift and the off-diagonal turning-shift POCs have been given in the first part of the present paper). In order to determine 9 principal POCs π_{im} (i, m = 1, 2, 3), one should therefore realize 9 independent experimental geometries, which would be described by 9 equations of the type of

$$\delta\Delta n_k = -\frac{1}{2}(\pi_{im} n_i^3 - \pi_{jm} n_j^3)\sigma_m,$$

three of which refer to 45°-directions of the polarization and the light propagation. Here Δn_k denotes the birefringence, π_{im} and π_{jm} the absolute POCs, σ_m the mechanical stress, index k marks the light propagation direction, i and j the polarization directions within the plane perpendicular to the k direction, and m the direction of the uniaxial pressure. This subject has been in detail considered in [22,30-32]. The three other POCs π_{44}, π_{55} and π_{66} are described by the equations for birefringence, which contain, besides of the POCs themselves, some complicated combinations of the principal π_{im} [22,30]. As a result, those POCs cannot be determined with a high accuracy.

The most commonly used method for studying the absolute POCs is still interferometry (see [22] for the past and [33,34] for the present-day techniques). Eventually, the same also refers to the absolute electrooptic coefficients [35]. But the absence, until the recent times, of the complete description of POE with respect to the elasticity effect has hindered the applicability of this method. The remarks about the applications of interferometers to the studies of POE, including those of a principle, will be formulated in detail at the beginning of chapter 6 of the present article.

A rich bibliography of the works on piezooptics in crystals until 1983 is contained, e.g., in [22,36-38].

6. Analysis of results for the absolute piezooptic coefficients

In the first part of the present article [24], the methodology of determination of the absolute POC has been led to the working formulae. These formulae give a possibility to complete the POC matrices for the crystals of all symmetry classes. Some examples of all the determined independent POCs for the crystals of different symmetry classes are brought together in Table 1. As one can see, a number of studies, where the problem of completing the POC matrix is in full solved, reduces with lowering the symmetry class of crystal, i.e. with a complication of the POC matrix. In many cases of piezooptic studies, the authors do not accentuate at all whether they take the elastic deformation of sample into account when determining the POC, or not. There is therefore a ground to object the facts reported there. There are also the works where the formulae for π_{im} are used, in which the multiplier n_i is present in the elastic term, instead of $n_i - 1$. This represents another source of flagrant error in the calculations of π_{im}. The deficiency is typical, for example, for the monograph [22] and it is apparently carried from the book [39] of the ancestor of modern piezooptics F.Pockels, to whom the author [22] refers in many cases. Probably, it has a sense to cite some curious examples of the studies of π_{im}. So, the work [40] reports the results for the absolute π_{im} in taurine crystal, which belongs to the monoclinic system, giving the values of all 20 independent POCs. However, the article does not contain the

Table 1. Absolute POCs of some crystals (in Br≡10^{-12} m^2/N).

Non-zero independent POCs Crystal (symmetry class)	π_{11}	π_{12}	π_{13}	π_{31}	π_{33}	π_{14}	π_{41}	π_{44}	π_{55}	π_{66}
Alum $KAl(SO_4)_2 \cdot 12H_2O$ +0,6%Cr (m3) [51]	1.8	8.3	5.7	π_{12}	π_{11}	–	–	–0.1	π_{44}	π_{44}
Barium-strontium niobate (Ba_xSr_{1-x}) $\times Nb_2O_6$, x=0.4 (4mm) [52]	–2.5	–2.9	–2.0	–3.8	4.7	–	–	–3.6	π_{44}	3.6
$LiNbO_3$ (3m) [44]	–0.47	0.11	2.0	0.47	1.6	0.7	–1.9	0.21	π_{44}	$\pi_{11}-\pi_{12}$
$LiTaO_3$ (3m) [53]	–0.62	0.34	0.64	0.43	–0.07	0.40	0.07	0.41	π_{44}	$\pi_{11}-\pi_{12}$
Barium beta-borate β–BaB_2O_4 (3m) [43]	–1.7	–1.35	1.75	–1.6	3.7	–2.0	–2.0	–26	π_{44}	$\pi_{11}-\pi_{12}$
Roshelle salt (222) [30]	3.1 π_{22}= =0.76	3.4 π_{21}= =5.4	3.2 π_{23}= =2.1	5.0 π_{32}= =2.9	1.6	–	–	–2.5	1.5	–2.5
Cs_2HgCl_4 (mmm) [48,49]	2.3 π_{22}= =11.8	17.5 π_{21}= =–1.4	14,8 π_{23}= =16.6	–1.7 π_{32}= =7.3	13.2	–	–	–11	–5.8	–10.4

working formulae, which should have been complicated enough [24], owing to a presence in the POC matrices of eleven non-principal independent POCs, but instead there is a qualitative reference to the Pockels' method only. As a consequence, there appears a confidence, rather than simply a doubt, that the results of this work are not true. We notice also that, until the appearance of the works [24,41,42] (all performed with the participation of the author of this review), one could not find in the literature any references, containing the working formulae for determination of the non-principal POCs for the crystals with a symmetry lower than the cubic one. Only the work [22] gives the correct formulae for π_{14}, π_{41} and π_{44} for the most simple, with respect to the symmetry, cases of trigonal crystals (the symmetry classes of 3m, $\bar{3}$m and 32), though without any indication at the sign dualism of these POCs and the necessity to fix the signs of axes of the right-hand coordinate system [24,44-46], and again with the above-mentioned error in the elastic term (the multiplier n_i instead of $(n_i - 1)$).

We should also point at the work [37], where the working relations are given for the cubic and simpler-symmetry trigonal crystals, but without the sign dualism for the coefficients π_{14}, π_{41} and π_{44} and without the elastic term. The deficiencies of the POE studies mentioned above may probably explain the essential lack of agreement between the values of the π_{im} coefficients for $LiNbO_3$ crystals determined in [45] and [47].

Thus, having taken the above reservations into account, it is necessary to call in question the results of the most of works, in which all the absolute POCs π_{im} are determined. Therefore, Table 1 includes the π_{im} values for the crystals studied with participation of the author of this review, as well as the results [48,49], which are familiar for the author, and those of [30], where the correct working relations are written out and the careful experiments performed. This means that the values of π_{im} are obtained in [30] with a number of methods: (1) the retardations induced with the uniaxial pressure are measured, using the polarization-optical method (a rhombic symmetry of the Roshelle salt crystals under study allows to calculate all of the absolute coefficients π_{im} after these data – see Table 5.2 in [22]), (2) some values of π_{im} are found directly with the interferometric method, and (3) the relations for some elastooptic coefficients (like for the p_{21}/p_{11} and p_{13}/p_{33} ratios) are found with the acoustooptic diffraction method, which are subsequently compared with the coefficients p_{im} calculated on the basis of π_{im}. All the results are well coordinated (within the limit of 10%), thus giving a ground to consider them as being unbiassed. By the way, they agree in the main with the results of the latest works [31,50], too.

On the basis of the completed POC matrices (Table 1), below we give the examples of construction of POE indicative surfaces (or the so-called piezooptic surfaces), their sections by the principal planes and also stereographic projections of these surfaces, which demonstrate in the most complete manner both the value and the anisotropy of POE in crystals.

7. Indicative surfaces of piezooptic effect for trigonal and cubic crystals

The indicative surface (IS) is a spatial surface described with *radius-vector of a physical effect* [54-56]. The value of the radius-vector ($\vec{\mathbf{R}}$) along the given direction is equal to the value of the physical effect in the same direction. The direction $\vec{\mathbf{R}}$ is defined in space by the direction cosines of the angles between $\vec{\mathbf{R}}$ and the axes X_1, X_2 and X_3 of crystal-physical coordinate system, or by the spherical coordinates θ and φ.

After citing the definitions, we emphasize that the IS mathematically corresponds to the transformation law for the components of tensor of the given physical effect, under transiting from the "old" (crystal-physical) coordinate system X_1, X_2, X_3 to the "new" (or "mobile") one X'_1, X'_2, X'_3. So, the IS for the piezooptic effect, which is described by a rank-four tensor π_{im}, looks as

$$\begin{aligned}\vec{\mathbf{R}}(\alpha_{mi},\alpha_{nj},\alpha_{pk},\alpha_{gl}) \equiv \pi'_{mnpg} = \\ = \alpha_{mi}\cdot\alpha_{nj}\cdot\alpha_{pk}\cdot\alpha_{gl}\pi_{ijkl}\end{aligned}, \qquad (1)$$

where π'_{mnpg} is the indicative surface, π_{ijkl} the components of piezooptic tensor in the old (crystallographic) coordinate system, and $\alpha_{mi},\ldots,\alpha_{gl}$ the direction cosines between the axes of the new and the old coordinate systems.

Trigonal crystals

Let us write down formula (1) in the detailed form for the case of niobate crystals belonging to the trigonal symmetry class 3m, and take into account the corresponding POC matrix containing only eight independent POCs π_{im} (see Table 1). This will take into account the known dependences of POCs written in the matrix (π_{im}) and tensor (π_{ijkl}) forms ($\pi_{im} = \pi_{ijkl}$ at $m = 1,2,3$ and $\pi_{im} = 2\pi_{ijkl}$ at $m = 4,5,6$).

For the longitudinal component of the POE tensor π'_{11} referred to as the "longitudinal POE" later on (this component describes a change in the birefringence along the axis X'_1 under the action of uniaxial pressure in the same direction) we get:

$$\begin{aligned}\pi'_{11} = \pi_{11}(\alpha_{11}^2 + \pi'_{11} = \pi_{11}(\alpha_{11}^2 + \alpha_{12}^2)^2 + \\ +(\pi_{14} + 2\pi_{41})(3\alpha_{11}^2 - \alpha_{12}^2)\alpha_{12}\alpha_{13} + \\ +\pi_{33}\alpha_{13}^4 + (\pi_{13} + \pi_{31} + 2\pi_{44})\times(\alpha_{11}^2 + \alpha_{12}^2)\alpha_{13}^2.\end{aligned} \qquad (2)$$

For the transverse component π'_{12} of the POE tensor referred to as the "transverse POE" (it describes a change in the birefringence along the axis X' under the action of uniaxial pressure along the axis X'_2) we have:

$$\begin{aligned}\pi'_{12} = \pi_{11}(\alpha_{11}\alpha_{21} + \alpha_{12}\alpha_{22})^2 + \pi_{12}(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})^2\alpha_{23}^2 + \pi_{13}(\alpha_{11}^2 + \alpha_{12}^2)^2\alpha_{23}^2 + \pi_{33}\alpha_{13}^2\alpha_{23}^2 + \\ +\pi_{31}(\alpha_{21}^2 + \alpha_{22}^2)^2\alpha_{13}^2 + \pi_{14}(\alpha_{11}^2\alpha_{22} - \alpha_{12}^2\alpha_{22} + 2\alpha_{11}\alpha_{22}\alpha_{21})\alpha_{23} + 2\pi_{41}(\alpha_{12}\alpha_{22}^2 - \alpha_{12}\alpha_{22}^2 + \\ +2\alpha_{11}\alpha_{21}\alpha_{22})\alpha_{13} + 2\pi_{44}(\alpha_{11}\alpha_{21} + \alpha_{12}\alpha_{22})\alpha_{13}\alpha_{23}.\end{aligned} \qquad (3)$$

To construct the indicative surfaces (2) and (3), we should pass on to the spherical coordinate system, and then express the direction cosines in terms of the spherical coordinates θ and φ.

In case of the longitudinal POE it is enough to set the direction of the axis X'_1, along which the pressure acts and the light is polarized. That is why the radius-vector $\vec{\mathbf{R}} \parallel X'_1$ is determined by the three direction cosines α_{11}, α_{12} and α_{13}, for which the known relations give

$$\begin{aligned} &\alpha_{11} = \sin\theta\cos\varphi; \\ &\alpha_{12} = \sin\theta\sin\varphi; \qquad (4) \\ &\alpha_{13} = \cos\theta. \end{aligned}$$

Inserting (4) into (2), we arrive at the IS for the longitudinal POE:

$$\begin{aligned} &\pi'_{11} = \pi_{11}\sin^4\theta + \pi_{33}\cos^4\theta + \\ &+\frac{1}{4}(\pi_{13} + \pi_{31} + 2\pi_{44})(\sin 2\theta)^2 + \qquad (5) \\ &+(\pi_{14} + 2\pi_{41})\sin^3\theta\cos\theta\sin 3\varphi. \end{aligned}$$

To obtain the sections of the surface (5) with the principal planes, it is necessary to insert into (5) the values of θ and φ, which correspond to these planes, i.e., the sections by planes $\perp X_1$, $\perp X_2$ i $\perp X_3$ are determined by trivial conditions φ=0°, φ=90° and θ=90°, respectively. Inserting these angles into (5), we obtain the following expressions for the principal sections of the IS:

$$\begin{aligned} &\pi'_{11}(\perp X_1) = \pi_{11}\sin^4\theta + \pi_{33}\cos^4\theta + \\ &+\frac{1}{4}(\pi_{13} + \pi_{31} + 2\pi_{44}), \qquad (6) \end{aligned}$$

$$\begin{aligned} &\pi'_{11}(\perp X_2) = \pi_{11}\sin^4\theta + \pi_{33}\cos^4\theta + \\ &+\frac{1}{4}(\pi_{13} + \pi_{31} + 2\pi_{44})(\sin 2\theta)^2 + \qquad (7) \\ &+(\pi_{14} + 2\pi_{41})\sin^3\theta\cos\theta, \end{aligned}$$

$$\pi'_{11}\ (\perp X_3) = \pi_{11}\ \text{(this is a circle)} \quad (8)$$

For the longitudinal POE described by equation (3) we are to point out the following typical differences: firstly, it is necessary to set the two directions (those of the light polarization (i) and the uniaxial pressure (m)) rather than one, and, secondly, the IS of the diametrical POE should be constructed depending on whether (1) the radius-vector $\vec{\mathbf{R}}$ coincides with the polarization direction, or (2) $\vec{\mathbf{R}}$ coincides with the pressure direction.

In the first case of the piezooptic surface construction, the pressure direction $m \parallel X'_2$ can change in the isotropic plane X_1X_2 perpendicular to the optical axis, and the polarization direction $i \parallel \vec{\mathbf{R}} \parallel X'_1$ for each "fixed" direction of m changes in the plane perpendicular to m, i.e., in the plane $X'_1X'_3$. Let us note that the conditions $i \parallel X'_1$ and $m \parallel X'_2$ are chosen far not accidentally – they correspond to determination of component π'_{12} of the POE tensor in the coordinate system X'_1, X'_2,X'_3.

Now it is easy to obtain the values of the direction cosines for the each case in terms of spherical coordinates θ and φ for the directions of polarization $i \parallel \vec{\mathbf{R}}$ and pressure $m \perp \vec{\mathbf{R}}_{\text{pr}}$ (see Fig. 1, where the direction of pressure (m) or light polarization (i) is taken within the isotropic plane X_1X_2 and $\vec{\mathbf{R}}_{\text{pr}}$ is the projection of the radius-vector onto this plane):

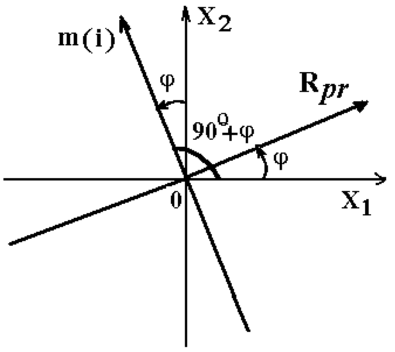


Fig. 1. Schematic representation of the directions of uniaxial pressure m (or light polarization i) and the radius-vector $\vec{\mathbf{R}}_{\text{pr}}$ projection within the isotropic plane X_10X_2 in optically uniaxial crystals.

$$\begin{aligned} &\alpha_{11} = \sin\theta\cos\varphi; && \alpha_{12} = \sin\theta\sin\varphi; \\ &\alpha_{13} = \cos\theta; && \alpha_{21} = -\sin\varphi; \qquad (9) \\ &\alpha_{22} = \cos\varphi; && \alpha_{23} = 0. \end{aligned}$$

In the second case of the surface construction for the transverse component π'_{12} of the POE tensor, the polarization direction $i \parallel X'_1$ changes within the plane X_1X_2, and the pressure direction $m \parallel \vec{\mathbf{R}} \parallel X'_2$ for each “fixed” direction of polarization i (within the plane perpendicular to i). Then we obtain for the directions $m \parallel \vec{\mathbf{R}}$ and $i \perp \vec{\mathbf{R}}_{\mathrm{pr}}$ (Fig. 1):

$$\begin{aligned}&\alpha_{11} = -\sin\varphi, \quad \alpha_{12} = \cos\varphi, \quad \alpha_{13} = 0,\\ &\alpha_{21} = \sin\theta\cos\varphi, \quad \alpha_{22} = \sin\theta\sin\varphi, \\ &\alpha_{23} = \cos\theta.\end{aligned} \qquad (10)$$

Inserting (9) and (10) into (3), one has the relations

$$\begin{aligned}\pi'^{(i)}_{12} &= \pi_{12}\sin^2\theta + \pi_{31}\cos^2\theta - \\ &\quad -\pi_{41}\sin 2\theta \sin 3\varphi,\end{aligned} \qquad (11)$$

$$\begin{aligned}\pi'^{(m)}_{12} &= \pi_{12}\sin^2\theta + \pi_{13}\cos^2\theta - \\ &\quad -\frac{1}{2}\pi_{14}\sin 2\theta \sin 3\varphi,\end{aligned} \qquad (12)$$

which describe the IS of the diametrical POE in spherical coordinates.

We call the surface (11) as *the piezooptic IS of the polarization*, because it is described in space by the radius-vector $\vec{\mathbf{R}}$ coinciding with the light polarization direction, while the surface (12) as *the piezooptic IS of the mechanical stress*, because it is described by the radius-vector $\vec{\mathbf{R}}$ coinciding with the pressure direction.

Inserting the values of θ and φ into (11) and (12), which correspond to the sections of these surfaces by the principal planes, we obtain, analogously to (6)-(8), the following equations for the principal sections:

1) for the surface of $\pi'^{(i)}_{12}$

$$\pi'^{(i)}_{12}(\perp X_1) = \pi_{12}\sin^2\theta + \pi_{31}\cos^2\theta, \qquad (13)$$

$$\begin{aligned}\pi'^{(i)}_{12}(\perp X_2) &= \pi_{12}\sin^2\theta + \pi_{31}\cos^2\theta + \\ &\quad +\pi_{41}\sin 2\theta,\end{aligned} \qquad (14)$$

$$\pi'^{(i)}_{12}(\perp X_3) = \pi_{12} \qquad (15)$$

(this is the circle);

2) for the surface of $\pi'^{(m)}_{12}$

$$\pi'^{(m)}_{12}(\perp X_1) = \pi_{12}\sin^2\theta + \pi_{13}\cos^2\theta, \qquad (16)$$

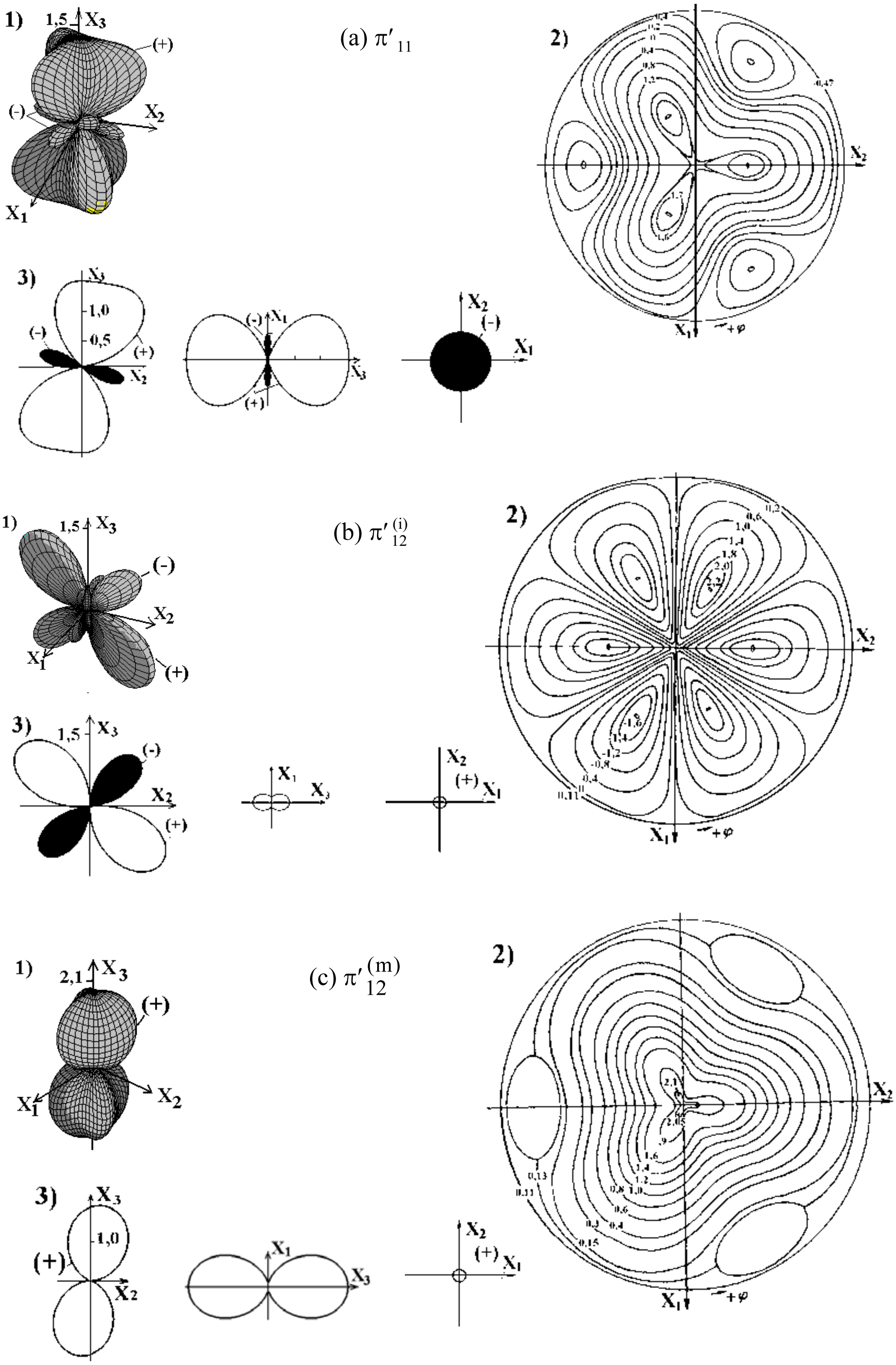
$$\begin{aligned}\pi'^{(m)}_{12}(\perp X_2) &= \pi_{12}\sin^2\theta + \pi_{13}\cos^2\theta + \\ &\quad +\frac{1}{2}\pi_{14}\sin 2\theta,\end{aligned} \qquad (17)$$

$$\pi'^{(m)}_{12}(\perp X_3) = \pi_{12} \text{ (this is a circle).} \qquad (18)$$

One can see that all the formulae (5), (11)-(12) for the IS of the longitudinal and transverse POE, along with formulae (6)-(8) and (13)-(18) for the sections of these ISs by the principal planes, are simple enough. That is why, a standard computer software would allow one to build, without any difficulties, all the corresponding ISs and their sections, to find the extremum (maximum and minimum) or some given values and their angular coordinates (θ, φ), to construct their stereographical projections, which provide complete quantitative characteristics of the POE anisotropy (see below). The examples of the ISs, their sections by the principal planes and stereographic projections of the ISs for the crystals of $LiNbO_3$ are demonstrated in Fig. 2. The analysis of these surfaces will be conducted at the end of the chapter. Here we restrict ourselves to the following worthwhile remarks only:

(1) The construction of the piezooptic IS π'_{12} is limited by the modes described above, when the “fixed” direction m (or i), around which the radius-vector $\vec{\mathbf{R}}$ rotates, can change within the principal plane only, i.e., perpendicularly to the optical axis. The reason is that, for the light propagating in the anisotropic direction k in crystal, there are two possible (mutually perpendicular) polarization directions only, which lie in the plane perpendicular to the direction k, i.e., one of the polarizations is still placed in the isotropic plane. Therefore, only the above-described ways for the POE IS construction may ensure a reciprocal perpendicularity of the three directions i, m and k, the only condition being consistent with the allowable geometries of piezooptic experiments. If the “fixed” direction m (or i) changed in any other non-isotropic plane, then the analysis of the construction procedure would demonstrate that it is impossible to satisfy the condition

$i \perp m \perp k$. In this respect, the POE is qualitatively different from the other effects described by fourth-rank tensors (for instance, the elasticity or the piezo-resistance effect), for which all the ex-


(a) π'_{11}

(b) $\pi'^{(i)}_{12}$

(c) $\pi'^{(m)}_{12}$

Fig. 2. Indicative surfaces of POE in $LiNbO_3$ crystals: a) the longitudinal POE (π'_{11}), the transverse POE ($\pi'^{(i)}_{12}$), and the transverse POE ($\pi'^{(m)}_{12}$). (1), (2) and (3) represent, respectively, the appearance of the IS, its stereographic projection and section by the principal planes.

perimental geometries are allowable (and, correspondingly, all the ways of the IS construction are allowable, when it is not obligatory to limit oneself to the isotropic plane for the possible directions of one of the axes X'_j). Therefore, these effects, which are characterized with the two qualitatively different IS π'_{12} only, allow, in principle, to construct an unlimited number of ISs for the transverse components of the corresponding tensor, contrary to the POE. This implies that an unambiguous comparative analysis of such the physical effects is quite difficult. As for the IS of the piezooptic effect, a limited number of the ISs confirms unambiguity and an unbiased character of both the relevant studies and implementation of the corresponding results.

(2) One can demonstrate that the surfaces $\pi'_{11}\equiv\pi'_{22}\equiv\pi'_{33}$ and $\pi'_{12}\equiv\pi'_{21}\equiv\pi'_{31}\equiv\pi'_{13}\equiv\pi'_{23}\equiv\pi'_{32}$, and therefore the equations (5), (11) and (12), describe completely the longitudinal ($i\,||\,m$) and transverse ($i\perp m$) POEs (for i, $m \le 3$) in the crystals of symmetry 3m (the same refers to the symmetry classes $\bar{3}$m and 32, which have the same POK matrices).

(3) It is necessary to construct the IS in the same coordinate system, where the POC π_{im} are measured (with taking the positive directions of the axes X_1, X_2, X_3 into account), in order to eliminate the errors related to ambiguity of the choice of coordinate system [24,44- 46].

(4) We emphasize that the surfaces π'_{11}, $\pi'^{(i)}_{12}$ and $\pi'^{(m)}_{12}$ correspond to the principal components of the POC matrix (i, m = 1,2,3) and describe the changes in the refraction index along the direction denoted by the first subscript, under the action of a uniaxial pressure (tension) applied along the direction denoted with the second subscript. We do not consider the surfaces π'_{im} with the indices i, m >3, because they have a rather educational character, though are of a little practical use in the light modulation, photoelastic pressure sensors, acoustic-optical devices, etc.

Cubic crystals

Since the POC matrices for cubic crystals are simpler in comparison with trigonal crystals (see Table 1), the equations for these ISs are simpler, too. The methodology for derivation of the IS equations is already considered above. That is why, here we confine ourselves to writing down the IS equations of the type of (5), (11) and (12) for cubic crystals, which may be divided into the two groups, according to their POC matrices: (1) the crystals of lower symmetry, belonging to the classes 23 and m3 (four independent POCs π_{11}, π_{12}, π_{13} and π_{44}), and (2) the crystals of higher symmetry described by the classes 432, $\bar{4}$3m and m3m (three independent POC π_{11}, π_{12} and π_{44}).

The formulae for the IS in case of the first-group crystals are as follows:

$$\begin{aligned}\pi'_{11} &= \pi_{11} + (\pi_{12} + \pi_{13} + 2\pi_{44} - 2\pi_{11})\times \\ &\times(\sin^2\theta\sin^2\varphi\cos^2\varphi + \cos^2\theta)\sin^2\theta,\end{aligned} \tag{19}$$

$$\begin{aligned}\pi'^{(i)}_{12} &= \frac{1}{2}(\pi_{11} - \pi_{44})\sin^2\theta(\sin 2\varphi)^2 + \\ &+\pi_{12}(\sin^2\theta\cos^4\varphi + \cos^2\theta\sin^2\varphi) + \\ &+\pi_{13}(\sin^2\theta\sin^4\varphi + \cos^2\theta\cos^2\varphi),\end{aligned} \tag{20}$$

$$\begin{aligned}\pi'^{(m)}_{12} &= \frac{1}{2}(\pi_{11} - \pi_{44})\sin^2\theta(\sin 2\varphi)^2 + \\ &+\pi_{12}(\cos^2\theta\cos^2\varphi + \sin^2\theta\sin^4\varphi) + \\ &+\pi_{13}(\cos^2\theta\sin^2\varphi + \cos^2\theta\cos^4\varphi).\end{aligned} \tag{21}$$

The analysis testifies that the $\pi'^{(m)}_{12}$ IS remains analogous to that for $\pi'^{(i)}_{12}$, but is rotated around the X_3 axis by 90°. This is why, Fig. 3 shows only an outward appearance of one of these ISs.

For the second group of cubic crystals we have the POCs π_{12}=π_{13} and so the IS formulae are simpler:

$$\begin{aligned}\pi'_{11} &= \pi_{11} + 2(\pi_{12} + \pi_{44} - \pi_{11})\times \\ &\times(\sin^2\theta\sin^2\varphi\cos^2\varphi + \cos^2\theta)\sin^2\theta,\end{aligned} \tag{22}$$

$$\pi'^{(i)}_{12} = \pi'^{(m)}_{12} = \pi_{12} + \frac{1}{2}(\pi_{11} - \pi_{12} - \pi_{44}) \times \times \sin^2\theta(\sin^2 2\varphi). \tag{23}$$

For optically isotropic media, one more condition is to be required: $\pi_{44} = \pi_{11} - \pi_{12}$. As a result, the analysis of (22) and (23) yields in the following quite simple expressions for the IS:

$$\pi'_{11} = \pi_{11} \quad \text{i} \quad \pi'^{(i)}_{12} \equiv \pi'^{(m)}_{12} = \pi_{12} \tag{24}$$

for all the values of θ and φ, i.e. formulae (24) describe spheres with the radii π_{11} and π_{12}, respectively, for the longitudinal and transverse POE.

8. Analysis of anisotropy of the piezooptic effect

We characterize the POE on the basis of both the equations and outward appearances of the indicative surfaces and their sections. Besides, it is easy to use stereographic projections of the IS for the quantitative analysis of the POE anisotropy. We demonstrate the essence of construction of those projections on the example of π'_{11} as a function of spherical coordinates (see, e.g., formula (5)). At first, we should find a set of values of the arguments θ, φ of function (5), which determine the directions, along which the function $\pi'_{11}(\theta, \varphi)$ has the same chosen values. Then, according to the rules for construction of stereographic projection of directions, we determine, for each i-direction defined by θ_i, φ_i, the coordinates x_{1i}, x_{2i} of their stereographic projection onto the diametral plane of projection X_1OX_2:

$$\begin{aligned} x_{1i} &= \tan(\theta_i/2)\cos\varphi_i, \\ x_{2i} &= \tan(\theta_i/2)\sin\varphi_i \end{aligned} . \tag{25}$$

A set of points at the plane X_1OX_2 constructed along this way describes one isoline for the chosen value of the function $\pi'_{11}(\theta,\varphi)$. Quite similarly, one can construct the other isolines for the chosen set of values of the IS $\pi'_{11}(\theta,\varphi)$. The construction of stereographic projections of the IS may be realized easy enough, when using a modern standard software. The examples of stereographic projections of the IS are given in Fig. 2 and 3 (see also [57-59]).

Now we summarize a brief analysis of the POE anisotropy:

1. For all the examples of the IS construction for the longitudinal and transverse POE brought together in Fig. 2 to 4, a conspicuous anisotropy is typical, including a possibility for inversion of sign of the effect.

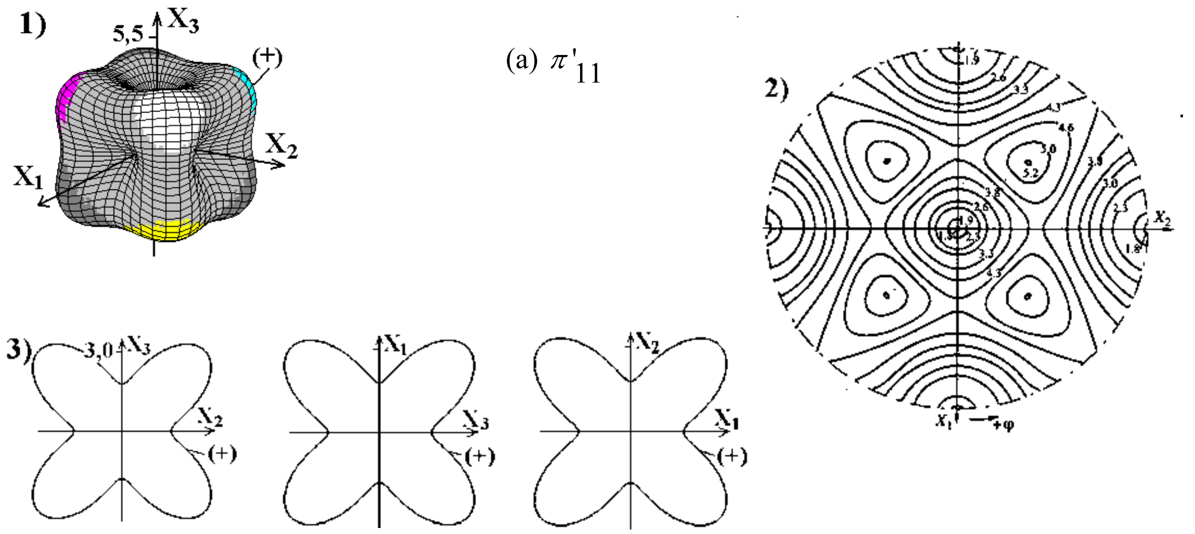
2. One can see from these figures that the ISs do not represent surfaces of rotation around the symmetry axes. This fact corresponds to the Hermann theorem [60]. According to it, the surfaces of rotation around the symmetry axis are the ISs of only those physical effects, whose tensors have a rank lower than the order of the mentioned symmetry axis. Hence, only in case of hexagonal crystals one can expect a simple form of the IS, i.e., a surface of rotation around the axis X_3 corresponding to the six-fold symmetry axis.

3. The ISs and their stereographic projections reveal all the symmetry elements characteristic of the crystal. Moreover, sometimes the ISs and the projections exhibit symmetry elements, which are not included in the point group of crystal. For example, we find a two-fold symmetry axis characteristic of the stereographic projections for $LiNbO_3$ crystals, which does not belong to the point group 3m. Similarly, in case of the longitudinal POE (the surface of π'_{11}) in the aluminium-potassium alum crystal we detect the four-fold axis directed along the second-order symmetry axis peculiar for the crystal itself.

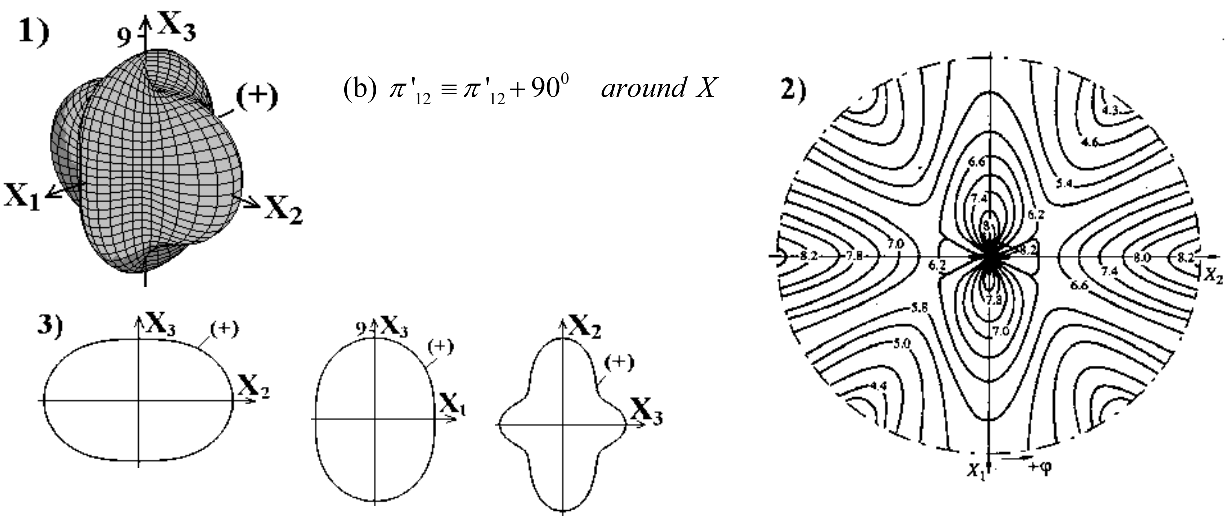
4. It looks quite unexpected a fact that, for all the cubic crystals, the extremum values of the longitudinal POE (the surface of π'_{11}) are observed along the same directions, independent of the relative values and signs of the POCs. This conclusion may be readily arrived at after inspecting the partial derivatives $\partial\pi'_{11}/\partial\theta$ and $\partial\pi'_{11}/\partial\varphi$ (see formulae (19) and (22)). For

instance, one of the extrema at the π'_{11} surface may be observed at $\theta = \arctan\sqrt{2} = 57{,}7^\circ$ and $\varphi = 45^\circ + n\times 90^\circ$ (n=0,1,2,3). Hence, we conclude that the π'_{11} surface shapes are alike for all cubic crystals. On the other hand, different signs and relative values of the POCs stipulate a variety of shapes of the POE surfaces, including a sign inversion for the extrema.

5. For the transverse POE (the π'_{12} surfaces) in cubic crystals, the orientation of extrema is different for the two groups of symmetry classes: 432, $\bar{4}3$m, m3m and 23, m3, respectively. While analyzing formula (23) in terms of partial derivatives for the first group, we find that the extrema have the orientation $\theta = 90^\circ$, $\varphi = 45^\circ + n\times 90^\circ$ (n=0,1,2,3) and $\varphi = 0^\circ$ or $\varphi = 90^\circ$ for the all values of θ (this corresponds to circular sections of π'_{12} surfaces by the planes $\perp X_1$ and $\perp X_2$; then we have $\pi'_{12} = \pi_{12}$). We did not succeed in determining the angular orientation of all the extrema for the second group (the classes 23, m3) with the aid of partial derivatives, since the expression (20) for π'_{12} is too complicated for this analysis. However, some extrema may be nevertheless found: e.g., the extrema of one of the groups are placed along the crystallographic axes X_1, X_2, X_3, and the second group is described by the coordinates



(a) π'_{11}



(b) $\pi'_{12} \equiv \pi'_{12} + 90^0$ *around X*

Fig. 3. Indicative surfaces of POE in $KAl(SO_4)_2{\cdot}H_2O$+0,6%Cr; crystals. The notations are the same as in Fig. 2.

$\theta = 90^\circ$ and $\varphi = \pm 52^\circ + n \times 180^\circ$ (n=0; 1).

6. There is an interesting peculiarity of the π'_{12} surface for the crystals with symmetries 23 and m3 (see Fig. 3). Here the IS crosses the axis X_3 not at a single point, but at a larger number of points lying at this axis within the interval from π_{12} to π_{13}. The explanation of this peculiarity has been made in [58].

7. For the trigonal crystals belonging to symmetry class 3m (e.g., $LiNbO_3$ [57] or $LiTaO_3$ [59]) one can also find coordinates of some extremum values of the IS with the same method of partial derivatives, applied to π'_{12} as a function of θ and φ. For example, the surface $\pi'^{(i)}_{12}$ for $LiNbO_3$ has three (the same in values) extrema along the directions $\varphi = 30^\circ + n \times 120^\circ$ (n=0,1,2), $\theta = 0{,}5\arctan[2\pi_{14}/(\pi_{12} - \pi_{31})] \approx 42^\circ$. In other words, the coordinates of extrema for the symmetry class 3m depend on the POC value, in contrast to the cubic crystals. Therefore, we expect here a larger variety of the IS shapes. As an example, a complete enough analysis for the IS differences in the crystals of lithium niobate and tantalate has been made in [59].

8. The coordinates of extrema can be also determined with a high accuracy from the stereographic projections of the ISs by means of stereographic reticle by Yu.V.Vulf. From the other side, having known the coordinates of the extremum (θ, φ), one can insert their values into relations for the surfaces π'_{11} and π'_{12} (see formulae (5), (11), (12), etc.), and so find the extremum values of the POE. For example, the maximum values of the POE for $LiNbO_3$ are equal to π'_{11} = 1,9 Br, $\pi'^{(i)}_{12}$ = 2.1 Br, whereas for the aluminium-potassium alum they are notably larger (π'_{11} = 5,3 Br, π'_{12} = 8.3 Br, etc.).

9. The IS may be also constructed with accounting for the elasticity effect. This problem is considered in [60].

More details about the essence and appearance of indicative surfaces for different physical effects may be found in [54-56,61-63], and for the POE in particular – in [45, 57-60].

Conclusion

In the present review we derive the equations and give the examples for the construction of piezooptic indicative surfaces for the cubic crystals and the trigonal crystals with simpler piezooptic matrices (the classes of 32, 3m and $\bar{3}$m). Besides, the formulae (5), (11) and (12) enable one to write out the equations for the piezooptic surfaces of hexagonal crystals with a relatively high symmetry (622, 6mm, 6/mmm and $\bar{6}$2m), after taking into account that π_{14} = $=\pi_{41}$= 0 for the mentioned crystals. The problem of construction of the IS is also solved for the tetragonal crystals belonging to the symmetry classes of 422, 4mm, 4/mmm and $\bar{4}$2m. The corresponding equations and the examples of the outward appearance of the IS are given in [64].

What concerns trigonal, tetragonal and hexagonal crystals with a lower symmetry (respectively, the classes 3 and $\bar{3}$, 4, $\bar{4}$ and 4/m, and 6, $\bar{6}$ and 6/m), we did not considered the

Fig. 4. Indicative surfaces of longitudinal POE in the rhombic crystals: a) Roshelle salt, and b) Cs_2HgCl_4 crystals.

subject, because the POC matrices have not been still completed for any crystal of the mentioned symmetry classes.

A separate remark should be made for optically biaxial crystals (i.e., rhombic, monoclinic and triclinic classes). The analysis of the ISs for those crystals demonstrates that their unambiguous construction is possible only for the longitudinal effect (the surface of π'_{11}). The example for such the construction for the Roshelle Salt and the crystals of Cs_2HgCl_4 may be seen in Fig. 4. When the directions of light propagation (*k*), polarization (*i*) and pressure (*m*) are required to be mutually perpendicular, the IS construction for the longitudinal piezooptic effect (the surfaces of π'_{12}) is problematic. If we even dropped the above condition of mutual perpendicularity, the IS construction would be anyway ambiguous, thus allowing no unbiassed analysis of the value and anisotropy of the POE.

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