Possible Principles of Optical 3D Tensor Stress Field Tomography

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Abstract

In the present paper the approach for solving the problem of 3D stress tensor field tomography is suggested. It is shown that the stress tensor field tomography can be based on the imaging polarimetry. The problem can be divided into three separate stages. In case of 2D stress distribution, one can easy obtain experimentally the distribution of the difference of stress tensor components (σ_1-σ_2) and the shear component σ_6. In case of 3D stress distribution, our approach is based on searching equi-stressed surfaces (if such the surfaces are non-closed) with the imaging polarimetry methods and using the rotation of sample in the index-matching liquid. Reconstruction of these surfaces allows one to reconstruct the 3D stress distribution in the sample. When the equi-stressed surfaces are closed, we suggest the cell model of the stressed medium and the approach based on the Jones matrices. We show that solving the system of 6*N* nonlinear equations of $(N)^{1/3}$ power with 6*N* variables (*N* being the number of cells, into which the stressed sample is divided) requires sample probing by a broad beam in $2(N)^{1/3}$ different directions.

Key words: optical tomography, imaging polarimetry, stress tensor

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Introduction

A need in a non-destructive reconstruction of 3D stress tensor field follows from such foreground directions as energy and materials saving, which depend on the quality and durability of constructions and constructive elements. It has lead to development of the methods for three-dimensional (3D) integrated photoelasticity [1,2]. These methods have demonstrated their efficiency in the determination of 3D stressed state for the samples with a high geometrical symmetry [2]. However, they cannot be applied for analyzing the models with the most general stressed state, as well as the semiconductor and dielectric elements. From the other hand, efficient analysis of 3D tensor stress fields could permit to design the constructions and constructive elements possessing a highest possible homogeneity, quality and mechanical durability, as well as minimal stresses and inhomogeneities. As for the elements of optoelectronics, this problem concerns first of all the growth of semiconductor and dielectric crystals, the lattice-matched semiconducting multi-layer structures [3], and the implanted layers, since they are known to possess in most cases the residual stresses and non-compatible deformations.

Recently a few theoretical approaches for solving the problems of the integrated photoelasticity in case of asymmetric objects have been proposed. The principal idea of these approaches consists in consideration of the reconstruction of 3D stresses from the point of view of optical tensor field tomography [4,5,6]. All of these approaches still remain to be only purely theoretical suggestions, which need further experimental verification. Unfortunately, it has not been done yet and, moreover, there are no precise experimental tools that could determine all the parameters necessary for the calculations. Moreover, all these approaches cannot be realized without some additional *a priori* information about the stressed state, or the numerical methods for solving the nonlinear equation systems would always lead only to a set of solutions otherwise.

A number of experimental techniques for determining three parameters (a characteristic retardation (2Δ), a primary (θ) and a secondary (γ) characteristic directions), which are usually used in the integrated photoelasticity, have been proposed [7,8]. All of these methods show a low accuracy; they are "slow" and so time-consuming . Thus, the optical data obtained by these methods cannot be used in the calculation algorithms as the initial parameters for stress reconstruction. Furthermore, one cannot obtain the data about the local optical indicatrix orientation inside a stressed body, when the optical indicatrix rotation around the direction of light propagation takes place.

In order to study the optical inhomogeneity, polarimeters of different kinds are used at present. Even if imaging polarimeters with a CCD-camera receivers are employed, the known approaches enable to obtain only a 2D stress distribution, and only if we know in advance that the stressed state is two-dimensional and the rotation of indicatrix around the optical path direction is absent. Concerning the stressed states of semiconductor objects, as far as we know, the imaging polarimetry has not yet been applied to the stress field reconstruction.

The present paper is devoted to search of the ways for solving all these problems in the case of 3D stress distribution in the initially isotropic transparent media.

Problem conditions

Principles and instrumentation of the scalar field optical tomography are now well developed [9]. They are based on solving the problem of 3D distribution of the scalar parameter, namely, the absorption of light. Scalar field X-ray and acoustic tomography are widely applied, for example, in medicine [10,11]. As scalars contain only one component for each of the N cells, in which the inhomogeneous medium is divided, the problem of finding, e.g., the absorption coefficient α_N, can be solved by measuring the integral absorption for different projections and solving the corresponding equation system. It is necessary to note that one can measure only one integrated component for each projection and so one should find only one component for each unit cell.

The polar vector field mapping already needs more complicated optical systems, which should be based on the nonlinear optical effects. For example, Y.Uesu suggested the SHG (second harmonic generation) microscope [12,13] in order for obtaining a 2D distribution of spontaneous polarization. In the case of tensor field tomography (we shall further consider a polar, symmetric second-rank tensor of impermeability coefficients B_{ij} at optical frequencies that describes the so-called optical indicatrix), each cell is in general characterized by six components. The tensor B_{ij} is linearly coupled to the second-rank polar stress tensor. But even in the case of homogeneously stressed state, it is possible to determine only three parameters of optical indicatrix after the one-projection measurements. If the incident beam is propagated along the z-axis of the optical indicatrix, one can usually determine polarimetrically the B_{12} component and the difference of components $(B_{11}-B_{22})$. If we use the interferometric methods, it is possible to determine B_{11} and B_{22} components separately ($B_{ij}=1/n^2$, with n being the refractive index). Moreover, such the measurements are correct only if the orientation of optical indicatrix does not depend on z coordinate, i.e. for a 2D distribution of inhomogeneity of the B_{ij}'s. Then the measurements for the other projections may allow to determine the three other parameters. It often leads to the situation when the number of the unknown parameters exceeds that of the nonlinear equations and the equation system can be solved neither analytically nor with a numerical methods. Hence, the problem lies in determination of the relation between the quantity of the projections needed and the complexity of the stress distribution.

Phenomenological approach

3D photoelastic specimens in general belong to the class of spatially non-uniform birefringent media. A typical property of such the media that makes them difficult for studies is a rotation of optical indicatrix around the light path direction. When indeed so, one cannot obtain the information about the local optical indicatrix orientation inside the stressed body and, as a consequence, the stresses inside the sample. Then it is only possible to apply the computer simulations which take into account the geometrical symmetry of the sample, the initial approximations about the stress field and the integrated polarimetric data.

This means that the following principal problems arise in designing an optical tomograph of stress tensor field:

- A necessity of possessing the additional information about the stressed state;
- Difficulties of obtaining the data on the local state of optical indicatrix, whenever the principal axes of the overall optical impermeability tensor rotate around the direction of light propagation.

Case 1(the sample is occupied by non-closed equi-stressed surfaces)

Let us assume that the sample under studies is transparent for the radiation used in the experiment and optically isotropic in its non-stressed state (accounting for the initial optical anisotropy can be done only after developing the algorithm for initially optically isotropic media). The position of each point inside the sample can be described by the coordinates *(x,y,z)* of the Cartesian coordinate system associated with the sample. If the additional information is absent, one can take into account only the obtained experimental data. Using the unambiguous experimental data from the overall 2D image, obtained for the light propagation through the sample in the chosen direction, one can consider the data related only for those elementary beams, for which a full extinction between the crossed polarizers exists and the mutual rotation of the crossed polarizers leads to the unity light modulation depth. For all the cells through which the elementary beam is propagated and for which the mentioned condition is satisfied, the equation for the optical indicatrices can be written as

$$\begin{aligned}&(B_1+\pi_{11}\sigma_1+\pi_{12}\sigma_2+\pi_{12}\sigma_3)x^2+\\&+(B_1+\pi_{12}\sigma_1+\pi_{11}\sigma_2+\pi_{12}\sigma_3)y^2+\\&+(B_1+\pi_{12}\sigma_1+\pi_{12}\sigma_2+\pi_{11}\sigma_3)z^2+\\&+2(\pi_{11}-\pi_{12})\sigma_4 zy+2(\pi_{11}-\pi_{12})\sigma_5 zx+\\&+2(\pi_{11}-\pi_{12})\sigma_4 xy=0\end{aligned}\quad ,\quad (1)$$

where B_i are the optical impermeability constants, π_{ij} the piezooptic coefficients, σ_j the components of the stress tensor (these components could in general depend on the three coordinates); and they would have an invariable orientation if the tensor σ_j along the given direction is constant. As it follows from the deformation-compatibility equation (the so-called Saint-Venant equation),

$$(e_{ij}^{(1)}-e_{ij}^{(2)})x_i x_j=0\,,\qquad (2)$$

(with $e_{ij}^{(1)},e_{ij}^{(2)}$ being the deformations of the neighbouring (“contacting”) regions of the continuous solid state) and Hook’s low,

$$e_{ij}=S_{ijkl}\sigma_{kl}\,,\qquad (3)$$

i.e. the surfaces of equal stresses in the solid state will be second-order surfaces. From the other hand, when the volume forces are absent, one can note on the basis of the elastostatics equation,

$$\operatorname{div}\sigma_{ij}=0\,,\qquad (4)$$

that these surfaces should be non-closed, i.e.

they could be neither spherical nor ellipsoidal. Such the non-closed second-order surfaces always contain straight generating lines. Thus, in the solid state which obeys the conditions mentioned above there always exist equi-stressed lines, the lines along which either stresses or the parameters of optical indicatrix should be constant.

If the wave vector normal coincides with the z direction of the coordinate system associated with the sample, it follows from the optical indicatrix equation that the angle of optical indicatrix orientation can be written for all the cells, through which the elementary beam is propagated, as

$$\tan 2\xi_z = \frac{2\sigma_6}{\sigma_1 - \sigma_2} = const(z) . \quad (5)$$

The latter relation will be useful for the next calculations. After measuring the optical retardation in the z direction, it is easy to calculate the stress tensor component difference (σ_1-σ_2) and the shear component σ_6 by solving the equation system

$$\begin{aligned}(\sigma_1 - \sigma_2) &= 2\Delta n_{12}\cos(2\xi_3)/n_o^3(\pi_{11} - \pi_{12}),\\ \sigma_6 &= \frac{1}{2}(\sigma_1 - \sigma_2)\tan(2\xi_3).\end{aligned} \quad (6)$$

Besides, the off-diagonal components of the integrated Jones matrix T_{ij}, which corresponds to the light propagation along such the direction, should be equal to zero. This could be quite easy checked experimentally[1]. The off-diagonal components of T_{ij} are zero in case if the off-diagonal components of all the elementary Jones matrixes t_{ij}, corresponding to the local elementary cells of this line, are equal to zero. Moreover, all the components of the local Jones matrixes t_{ij} will be the same for such the equi-stressed line and their product gives the components of the integrated Jones matrix T_{ij} .

Thus, the first step in the stress tensor field reconstruction lies in searching the equi-stressed surfaces, by means of measuring the extinction condition for different projections and rotating the sample in the index-matching liquid. The determined values of *(σ_1-σ_2)* and σ_{12} allow us to reconstruct σ_1, σ_2 and σ_{12} by the combined Muskhilishvili method [14]. Then the determined shapes of the equi-stressed surfaces enable reconstructing the 3D stress field distribution.

For instance, in case of a 2D stress distribution, there are no problems with determining the components σ_j and there is no need in dividing the sample into the elementary cells and calculating the Jones matrices. After measuring the optical indicatrix orientation angle and the optical retardation for the direction of constant stresses, it is easy to reconstruct the distribution of *(σ_1-σ_2)* and σ_6. Our results obtained by the imaging polarimetry technique for a glass disk loaded along the diameter are presented in Figure 1 and 2 as an example.

Thus, a 2D stress field can be reproduced by optical tomography methods, without any extra problems, mechanical models or initial approximations. [15].

<u>Case 2 (the sample is occupied by closed equi-stressed surfaces)</u>

The actions at this stage need a development of hybrid numerical-experimental methods for calculating a 3D stress field on the basis of maps for the optical anisotropy parameters obtained for different projections. The method of hybrid numerical-experimental reconstruction of a stress field consists in the following:

a) Let us choose a sample of cubic shape (divided into N cells, in order to simplify the presentation). Upon transmission of light along the directions coinciding with the cube edges, each elementary ray will transmit through $(N)^{1/3}$ cells, while the whole beam - through *(N)* cells.

The Jones matrix T_{ij} that describes the light transmission, along the chosen direction, through the whole sample, can be obtained by multiplying the local Jones matrixes t_{ij}

[1] For these beams, the full extinction between the crossed polarizers should exist and the mutual rotation of the crossed polarizers should lead to the unity light modulation depth. On the other hand, these beams should coincide with equi-stressed lines.

accounting for the separate elementary cells:

$$T_{ij} = \prod_{1}^{(N)^{1/3}} t_{ij} \, , \qquad (6)$$

where $(N)^{1/3}$ denotes the number of cells. Three components of each elementary Jones matrix t_{ij} are nonlinearly expressed in terms of the difference between the two diagonal components and the off-diagonal component of the "transverse" optical impermeability tensor. If a broad beam is transmitting in one direction, one can obtain $3(N)^{2/3}$ equations of $(N)^{1/3}$ power with $6N$ variables. It is therefore a necessary to obtain the corresponding maps for $2(N)^{1/3}$ different directions. As a result, a system of $6N$ nonlinear equations with $6N$ variables would be obtained. For example, in case of the sample with a cubic shape divided by 8 elementary cells, there is a need to provide the maps for 4 different directions, in case of 27 elementary cells – for 6 direction, in case of 225 elementary cubic cells – for 10 directions, and in case of 1000 elementary cells – for 20 directions. It is interesting to note that for the cubic sample with the dimensions of 1×1×1 cm^3 the volume resolution of 1mm^3 can be provided by 20 maps.

Basing on the given shape of sample and the necessity of filling up its whole volume by the same elementary cells, one can choose the shape of the elementary cells. It is interesting to notice that the shape of such the elementary cells could be the same as for the crystallographic elementary cells.

b) This stage consist in solving numerically (e.g., with the iterative method) the system of

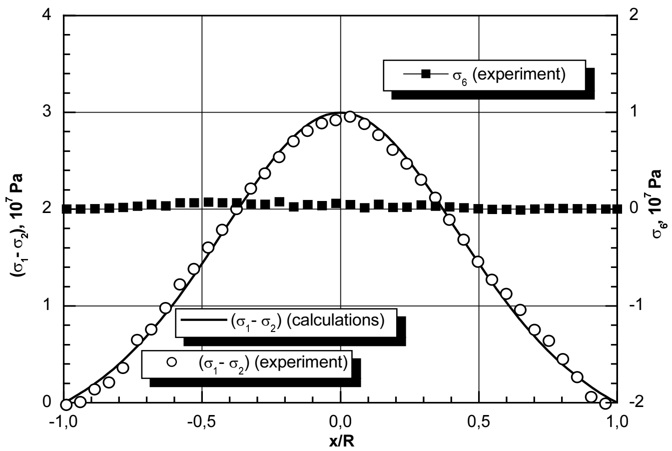

Fig.1. Stress distribution along a diameter of glass disk.

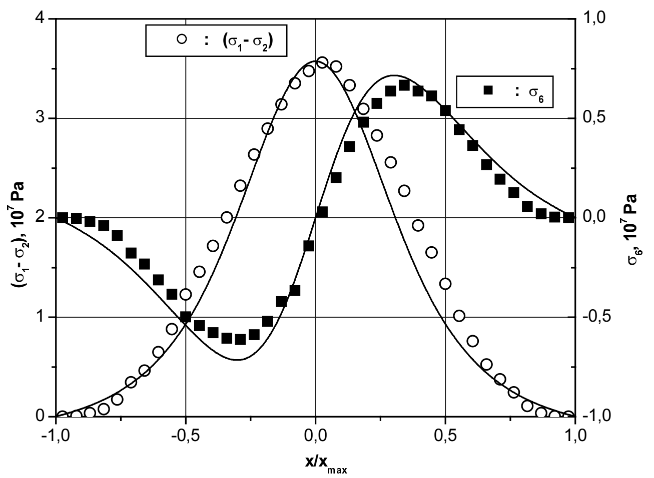

Fig.2. Stress distribution along a chord of glass disk.

the nonlinear equations

$$\begin{cases} T^{(1)}{}_{ij} = \prod_{n=1}^{N^{1/3}} t_{ij}^{(1)} \\ T^{(6N)}{}_{ij} = \prod_{n=1}^{N^{1/3}} t_{ij}^{(6N)} \end{cases} \quad (7)$$

using the initial approximations obtained at the first stage and the corresponding data (the integrated Jones matrices are calculated while using the retardation δ, the orientation of eigen-polarizations θ and the ellipticity of the eigen-polarizations ε) derived from the polarimetric maps for different projections.

In general, each elementary Jones matrix t_{ij} is coupled with the combination of six components of B_{ij} tensor, namely with those components that determine the parameters of elliptical cross section of the optical indicatrix, perpendicular to the given beam direction. For example, the components of n^{th} elementary Jones matrix are coupled to the components of optical impermeability tensor of n^{th} cell, written in the proper coordinate system, as

$$\begin{aligned} t_{11}^n &= \frac{1}{l_2^n - l_1^n}\left[l_1^n \exp(Cs_1^n d^n) + l_2^n \exp(Cs_2^n d^n)\right], \\ t_{22}^n &= \frac{1}{l_2^n - l_1^n}\left[l_1^n \exp(Cs_1^n d^n) - l_2^n \exp(Cs_2^n d^n)\right], \\ t_{12}^n &= \frac{1}{l_2^n - l_1^n}\left[-l_1^n \exp(Cs_1^n d^n) - l_2^n \exp(Cs_2^n d^n)\right], \\ s_{1,2}^n &= \frac{B_{11}^n + B_{22}^n}{2} \pm \frac{\Delta}{2}, \qquad (8) \\ l_{1,2}^n &= \frac{2B_{12}^n}{(B_{11}^n - B_{22}^n) \pm \Delta}, \\ \Delta &= \left[(B_{11}^n - B_{22}^n)^2 + (4B_{12}^n)^2\right]^{\frac{1}{2}}, \\ C &= \frac{ik_0}{2n_0}. \end{aligned}$$

(n_0 being the refractive index and k_0 the wave vector), while the latter tensor components are coupled to the mechanical stress via the relations

$$\begin{aligned} B_{11}^n &= B_{11}^o + \pi_{11}\sigma_{11}^n + \pi_{12}\sigma_{22}^n + \pi_{12}\sigma_{33}^n, \\ B_{22}^n &= B_{11}^o + \pi_{12}\sigma_{11}^n + \pi_{11}\sigma_{22}^n + \pi_{12}\sigma_{33}^n, \qquad (9) \\ B_{12}^n &= B_{11}^o + (\pi_{11} - \pi_{12})\sigma_{12}^n, \end{aligned}$$

As a result, after solving the mentioned equations we shall obtain the three differences of the diagonal stress tensor components and the three off-diagonal stress tensor components for each of the *N* cells.

Conclusion

In the present paper the approach for solving the problem of 3D stress tensor field tomography is suggested. It is shown that the stress tensor field tomography can be based on the imaging polarimetry. The problem can be divided into three separate stages. When a 2D stress distribution is dealt with, one can easy obtain experimentally the distribution of the difference of stress tensor components $(\sigma_1$-$\sigma_2)$ along with the shear component σ_6. In case of a 3D stress distribution, our approach is based on searching for the equi-stressed surfaces (if such the surfaces are non-closed) by the imaging polarimetry methods and further rotating the sample in the index-matching liquid. A reconstruction of these surfaces allows reconstructing the 3D stress distribution in the sample. When the equi-stressed surfaces are closed, we suggest the cell model of the stressed medium and the approach based on the Jones matrix technique. It is shown that solving the system of *6N* nonlinear equations of $(N)^{1/3}$ power with *6N* variables (*N* being the number of cells, on which the stressed sample is divided) requires probing the sample by a broad beam in $2(N)^{1/3}$ different directions.

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