Methods for the Studies of the Piezo-Optical Effect in Crystals and the Analysis of Experimental Data

I. Methodology for the studies of piezo-optical effec

B.Mytsyk

Lviv Centre of Space Research Institute, 5a Naukova Str., 79000, Lviv-53, Ukraine

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Abstract

In this review article, the attempt is made to give, in a brief form, a methodology of studying a piezo-optical effect, beginning from the methods and basic relations for obtaining the absolute piezo-optical coefficients on the basis of experimental data and finishing with accounting for elastic deformation of sample and the rules for giving positive signs to the axes of the right crystal-physical coordinate system. To make the picture complete, the relations are also given for determination of the piezo-optical coefficients of retardation and birefringence, since they are important in the applied aspects. The article is dedicated mainly to the methods for deriving the working formulae. Considering a restricted volume of the article, we do not touch upon the experimental methodology and its peculiarities, with the only exception for the immersion-interferometric method, which is described in a sufficient detail.
The complete relations (i.e., the relations accounting the elasticity effect) for determining the absolute piezo-optical coefficients by means of interferometric methods are given in table 2, while the relations needed for determining them from the optical indicatrix rotation or the birefringence may be found in the text. The strict expressions for the piezo-optical coefficients of birefringence and retardation, based on the determination of optical polarization constants, are also given in the text. These relations and the appropriate comments give a possibility to study comprehensively the piezo-optical effect in crystals of all the symmetry classes (the remarks concerning the triclinic crystals see in **Conclusion**).

Key words: piezo-optical effect, elastic deformations, optical indicatrix.

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Content

Introduction

All the absolute piezo-optical coefficients can be measured on the basis of changes in the refractive indices under the action of mechanical stress. The only acceptable method for that is interferometry, though there is also some information about using the prism method for this purpose. However, nowaday utilization of lasers might convince each investigator in that the interferometry is much more convenient and precise than the prism method. Therefore the working formulae in this article are written for the interferometric methods. These formulae are given in table 2 for one-pass interferometers, e.g. the Mach-Zender one. For the double-pass interferometers, e.g. the Michaelson one, it is necessary to multiply the r.h.s. of the relations presented in table 2 by the factor “2”, since the optical ray passes two times through the investigated sample. In case when the Phisot interferometer is dealt with, the relations of table 2 also need some corrections: the unity that enters both the expressions like (n_i–1) or ($\sqrt{2}/\sqrt{B_i - B_j}$ -1) and the elastic term, should be removed. The relations used to determine some absolute piezo-optical coefficients on the basis of the induced rotations of optical indicatrix, the optical birefringence or the retardation, are also important in the comprehensive studies of piezo-optical effect in crystals.

1. Main ideas and terms of piezo–optics

Piezo-optical effect (POE) is the change in the *optical characteristics* (refractive index, birefringence or retardation) of a solid under the action of *mechanical stresses*.

Historically, the birefringence Δn induced by mechanical stress σ has been discovered in the isotropic bodies and cubic crystals (D.Brewster, 1815 and 1818 – see [1,2]):

$$\Delta n = C_\sigma \cdot \sigma \, . \tag{1}$$

This relation is called as the Brewster’s law. The coefficient C_σ is known under different terms: the Brewster’s constant, the relative piezo–optical coefficient, or the optical coefficient of stress.

To describe the POE in anisotropic media (crystals), F.Pockels [2] has suggested the tensor relation

$$\delta B_{ij} = \pi_{ijkl} \sigma_{kl}; \;\; i,j,k,l = 1,2,3 \;\; , \tag{2}$$

the correctness of which has been successfully confirmed by the entire empiric experience of that time. In formula (2), δB_{ij} denotes the change in a specific component of the optical impermeability tensor, referred to afterwards as the *tensor of polarization constants* B_{ij} (it represents a material tensor of a rank two), σ_{kl} are the components of the *tensor of mechanical stresses* (a field tensor of rank two), π_{ijkl} the components of the rank-four *tensor of piezo–optical coefficients* (POCs). The internal symmetry of tensors B_{ij} and σ_{kl} allows to write formula (2) in the *matrix form:*

$$\delta B_i = \pi_{im} \sigma_m \, , \tag{3}$$

where the indices i and m mean, respectively, the directions of light polarization and the uniaxial compressing–tension, and they take the values 1,2,3,…6. The coefficients π_{im} and π_{ijkl} are called *absolute piezo–optical coefficients*. The link between the coefficients π_{im} and π_{ijkl} is given by the following expressions [1]:

$$\begin{aligned} \pi_{im} &= \pi_{ijkl} \quad \text{for} \quad m = 1,2,3; \\ \pi_{im} &= 2\pi_{ijkl} \quad \text{for} \quad m = 4,5,6. \end{aligned} \tag{4}$$

Since the symmetric tensor of polarization constants B_i has six independent components δB_1, δB_2, …, δB_6, formula (3) represents six equations. Since, in its turn, the symmetric tensor of mechanical stresses σ_m has also six independent components σ_1, σ_2, …, σ_6, each of the equations (3) has six terms $\pi_{im}\,\sigma_m$. For example,

$$\begin{aligned} &\delta B_1 = \pi_{11}\sigma_1 + \pi_{12}\sigma_2 + \pi_{13}\sigma_3 + \pi_{14}\sigma_4 + \\ &+ \pi_{15}\sigma_5 + \pi_{16}\sigma_6 \, , \\ &\dots \\ &\delta B_6 = \pi_{61}\sigma_1 + \pi_{62}\sigma_2 + \pi_{63}\sigma_3 + \pi_{64}\sigma_4 + \\ &+ \pi_{65}\sigma_5 + \pi_{66}\sigma_6 \, . \end{aligned} \tag{5}$$

That is, the general *matrix* of absolute POCs π_{im} contains 36 independent non–zero components π_{im}. Let us write this POC matrix and give the definition of four typical groups of the coefficients π_{im}:

$$\begin{array}{cc} & \begin{array}{cc} \quad\mathrm{I} & \qquad\qquad\mathrm{II} \end{array} \\ \pi_{im} = & \left|\begin{array}{ccc:ccc} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} & \pi_{15} & \pi_{16} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} & \pi_{25} & \pi_{26} \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} & \pi_{35} & \pi_{36} \\ \hdashline \pi_{41} & \pi_{42} & \pi_{43} & \pi_{44} & \pi_{45} & \pi_{46} \\ \pi_{51} & \pi_{52} & \pi_{53} & \pi_{54} & \pi_{55} & \pi_{56} \\ \pi_{61} & \pi_{62} & \pi_{63} & \pi_{64} & \pi_{65} & \pi_{66} \end{array}\right| \\ & \begin{array}{cc} \quad\mathrm{III} & \qquad\qquad\mathrm{IV} \end{array} \end{array} \qquad (6)$$

The rows of this matrix correspond to the first index of π_{im} and the change in the corresponding polarization constants B_i (i= 1, 2, …, 6), while the columns correspond to the second index and the corresponding components of the mechanical stress tensor σ_m (m = 1,2,…6). The three first rows of the matrix (6) correspond to the components δB_i with i = 1, 2, 3, which describe the changes in the *principal* refractive indices n_i (so far as $B_i = 1/n_i^2$), and the three last lines correspond to the components δB_4, δB_5, δB_6, which describe the *rotation* of *optical indicatrix* around the principal axes of the latter X_1, X_2, X_3, respectively (this is proved in the chapter 2.2, see, e.g., formula (30)). The first three columns in (6) correspond to the components σ_1, σ_2, σ_3, which represent *the mechanical stresses of compressing (or tension);* they are also called *normal stresses*. The three last columns in (6) correspond to the components σ_4, σ_5, σ_6, which are called *the mechanical stresses of shift (tangential stresses)* in the planes perpendicular to the axes X_1, X_2, X_3, respectively. After these remarks, we introduce the definitions of different POC groups in formula (6) [3,4]:

group I – 9 *principal* POCs π_{im} (i, m =1, 2, 3), which describe the change in the principal refractive indices under the action of normal (compression–tension) mechanical stresses σ_m;

group II – we call these 9 coefficients π_{im} (i = 1, 2, 3, m = 4, 5, 6) as the *shift* coefficients, because they describe the changes in the principal n_i under the action of shift components of the tensor σ_m;

group III – these are 9 *turning* π_{im} (i = 4, 5, 6, m =1, 2, 3), which describe the rotations of optical indicatrix around the axes X_1, X_2, X_3 under the action of normal components of σ_m;

group IV – these are *turning–shift* POCs π_{im} (i, m = 4, 5, 6), and they describe the rotations of optical indicatrix around the axes X_1, X_2, X_3 under the action of shift components of σ_m.

Let us also notice that (6) is a matrix of the most general form and it defines the POE in the crystals of triclinic symmetry. The presence of any symmetry elements in crystals of higher symmetry causes some components of the matrix (6) to become zero or dependent on each other. That simplifies essentially the POC matrices. Their appearance for the all symmetry groups is cited in the majority of authoritative monographs on crystal physics, for example, in [1,5]. Besides, the *matrices of elastic coefficients* (see also [1,5]) are also necessary, in order to take into account the elasticity in the studies of POE.

Below we write out the helpful relation, which allows to define the value of the components and the appearance of *mechanical stress tensor* σ_m in the principal coordinate system X_1, X_2, X_3, if the vector $\vec{p}$ of a uniaxial pressure (with the module σ) is arbitrarily oriented with respect to X_1, X_2, X_3 and is given by the directional cosines a, b, c, i.e., $\overline{p} = \overline{p}(a,b,c)$. Then we have

$$\sigma_m = \left|\begin{array}{ccc} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{array}\right| \cdot\sigma$$

or, in a more compact form,

$$\begin{aligned} \sigma_m &= [\sigma_1,\sigma_2,\sigma_3,\sigma_4,\sigma_5,\sigma_6] = \\ &= \left[a^2, b^2, c^2, bc, ac, ab\right] \times \sigma. \end{aligned} \qquad (7)$$

It is clear from the relation (7) that $\sigma_1 = a^2 \cdot \sigma$; $\sigma_2 = b^2 \cdot \sigma$; $\sigma_4 = bc \cdot \sigma$, etc.

Particular example for using the relation (7). If a sample is oriented under the angle 45° with respect to the axes X_1 and X_2 (fig. 1) and the pressure with the value σ is applied along the diagonal between X_1 and X_2 (this direction is designated in fig. 6), then the directional cosines of the pressure vector $\vec{p} = \vec{p}(a,\ b,\ c)$ are as follows: $a = b = \cos 45° = \sqrt{2}/2$, $c = \cos 90° = 0$. Inserting these values into (7), we obtain:

$$\sigma_m = [1/2, 1/2, 0, 0, 0, 1/2] \times \sigma. \qquad (8)$$

In other words, the two normal components $\sigma_1 = \sigma_2 = \sigma/2$ and the shift component $\sigma_6 = \sigma/2$ exist in the coordinate system X_1, X_2, X_3. We shall use below the expression (7) to establish the relations for the definition of the shift, turning and the shift-turning POCs.

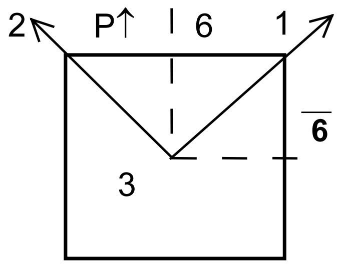


Fig.1. Action of uniaxial pressing–tension on a sample with 45^c – orientation with respect to the axes X_1, X_2.

In conclusion of this chapter, we write down the equations for *the "free" (non–perturbed* by the external mechanical influence*)* and the *perturbed* optical indicatrices in the coordinate system X_1, X_2, X_3. For the "free" indicatrix we get

$$B_1 x_1^2 + B_2 x_2^2 + B_3 x_3^2 = 1 \qquad \text{or}$$

$$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1, \qquad (9)$$

where $B_i = 1/n_i^2$ are the principal optical polarization constants, n_i the *principal* refractive indices (*the half-axis lengths of the optical indicatrix*), and x_i the coordinates.

Under the action of the stress σ_m, the principal components B_i change their values by δB_i (i = 1, 2, 3) and, moreover, the off–diagonal components of the polarization constant tensor δB_i (i = 4, 5, 6) appear, the latter defining the angles of inclination of the optical indicatrix with respect to the principal axes X_1, X_2, X_3 (see more details in chapter 2.2). Then we say that the optical indicatrix (the characteristic surface of the optical polarization constant tensor) is *perturbed*, and the corresponding equation looks as

$$\begin{aligned}&(B_1 + \delta B_1)x_1^2 + (B_2 + \delta B_2)x_2^2 + \\ &+ (B_3 + \delta B_3)x_3^2 + 2\delta B_4 x_2 x_3 + \\ &+ 2\delta B_5 x_1 x_3 + 2\delta B_6 x_1 x_2 = 1.\end{aligned} \qquad (10)$$

The expressions for δB_i for any experimental geometry may be derived on the basis of (3) or (5), after taking into account the non-zero components of the σ_m tensor defined by formula (7).

We denote the directions of crystal-physical axes X_1, X_2, X_3 and the corresponding directions of pressure (m), the light propagation (k) and the polarization (i) as 1, 2, 3; the diagonal directions between the axes X_2, X_3; X_1, X_3; X_1, X_2 as 4, 5, 6, respectively, and the directions perpendicular to them (one of the projections of the direction on the corresponding axes would be negative) as $\bar{4}$, $\bar{5}$, $\bar{6}$. Such the notation system for the directions is related to the Miller's notations through 1≡[100], 2≡[010], 3≡[001], 4≡[011], $\bar{4}$≡[0$\bar{1}$1]≡[01$\bar{1}$], 5≡[101], $\bar{5}$≡[$\bar{1}$01]≡[10$\bar{1}$], 6≡[110], $\bar{6}$≡[$\bar{1}$10]≡[1$\bar{1}$0]. The author of [1] also avoids the notations of the diagonal directions by the Miller indices. The notations used in this article are more laconic and, furthermore, they carry some elements of physical meaning. For example, the notation m=6 indicates that the component $\sigma_6 \neq 0$ (more exactly, $\sigma_6 = \sigma/2$ – see formula (8)), and it initiates non-zero POCs π_{i6}; the notation i=6 indicates that the non-zero coefficients π_{6m} are initiated, etc.

2. Absolute POCs: the relations for determination of all the POCs from the changes in the refractive indices and the rotation of optical indicatrix

2.1. Determination of the principal POCs with taking elasticity into account

The correct account for the elasticity effect is necessary for determination of the absolute POCs, since the influence of elastic deformation of the sample on the optical path change $n_i \cdot d_k$ under the action of mechanical stress σ_m could be considerable, sometimes even dominant (e.g., in the cases of POE, when the change in the optical path is mainly caused by the sample deformation, and the refractive index change $\delta n_i \to 0$).

Let us consider a crystal sample placed into the measuring arm of a one–pass interferometer (fig. 2), for example the Mach–Zender one [6,7]. One can see that the change in the optical path $\delta\Delta_i = \delta(n_i \cdot d_k)$ depends on both the refractive index change δn_i and the the sample thickness change δd_k:

$$\delta\Delta_k = \delta(n_i \cdot d_k) = \delta n_i \cdot d_k + \delta d_k \cdot n_i . \quad (11)$$

Besides, we have to take into account that the optical path decreases, because the crystal, being deformed by the value δd_k, subtracts from the optical path the value

$$\delta\Delta'_k = \delta d_k \cdot n_c , \quad (12)$$

where n_c is the refractive index of the medium in which the interferometer ray propagates.

If the sample is in the air, then n_c=1.

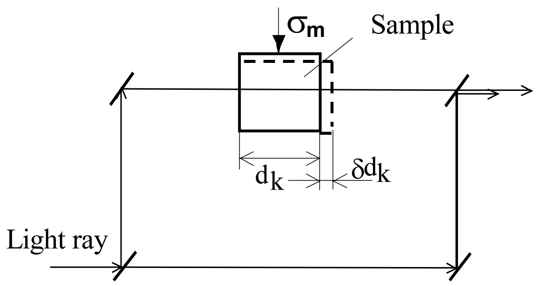


Fig.2. Sample deformation under the operation of σ_m: continuous line – $\sigma_m = 0$, dotted line - $\sigma_m \neq 0$.

Therefore, we rewrite (11) as:

$$\begin{aligned}\delta\Delta_k &= \delta n_i \cdot d_k + \delta d_k \cdot n_i - \delta d_k \cdot n_c = \\ &= \delta n_i \cdot d_k + \delta d_k (n_i - 1).\end{aligned} \quad (13)$$

The unity appearing in the brackets of the r.h.s. of equation (13) is often forgotten by workers in the field. This concerns also the initiator of piezo–optics in crystals, F.Pockels (see [8]). The mistake is included even in the contemporary earnest monograph [1].

Expressing δd_k in terms of the relative deformation e_k, we rewrite (13) in the form

$$\delta\Delta_k = \delta n_i \cdot d_k + e_k d_k (n_i - 1). \quad (14)$$

Formula (14) is central for a piezo-optical experiment. We can use it to write out the relation for determining the *principal* POCs π_{im} (*i, m* =1,2,3 – see *group 1* in (6)). Experimental determination of the principal POCs implies that the light polarization (*i*) and the uniaxial pressure (*m*) are directed along the principal crystallographic axes (the axes of the optical indicatrix X_1, X_2, X_3).

In the above case, it is easy to get the expressions for the changes in the refractive index δn_i induced by mechanical stress and the deformations e_k which enter equation (14). This may be done by means of the absolute POCs π_{im} (for δn_i) and the coefficients of elastic compliance S_{km} (for e_k). We obtain the expression for δn_i in terms of the principal components π_{im} from the polarization constants B_i and the equation (3) for piezo–optical effect.

According to definition, $B_i = 1/n_i^2$, resulting in the following relation between the small piezo-optical changes δn_i and δB_i:

$$\begin{aligned}\delta n_i &= \frac{1}{\sqrt{B_i + \delta B_i}} - \frac{1}{\sqrt{B_i}} = \\ &= -\frac{1}{2}\frac{\delta B_i}{B_i^{3/2}} = -\frac{1}{2}\delta B_i n_i^3 ,\end{aligned} \quad (15)$$

or, with taking (3) into account,

$$\delta n_i = -\frac{1}{2}\pi_{im}\sigma_m n_i^3 . \quad (16)$$

Now we can write the expression for e_k, which is given by the Hook's law:

$$e_k = S_{km}\sigma_m, \quad (17)$$

where S_{km} are the coefficients of elastic compliance.

Inserting (16) and (17) into (14), we obtain the relations for the principal POCs π_{im}, enabling one to determine them on the basis of experimental data for the optical path changes $\delta\Delta_k(\sigma_m)$, with accounting for the elasticity effect:

$$\delta\Delta_k = -\frac{1}{2}\pi_{im}\sigma_m d_k n_i^3 + S_{km}\sigma_m d_k (n_i - 1) \quad (18)$$

or $$\pi_{km} = \frac{2S_{km}}{n_i^3}(n_i - 1) - \frac{2\delta\Delta_k}{\sigma_m n_i^3 d_k}. \quad (19)$$

It is worthwhile that (16), (18) and (19) are strictly correct for the principal π_{im} in case when the turning coefficients π_{im} (*group III* in (6)) are equal to zero (that being typical for the crystals with a high symmetry and rhombic ones). Even if one of the mentioned coefficients is non-zero, then the corresponding component of the tensor σ_m also imposes a rotation of optical indicatrix, and the latter affects additionally the value δn_i. Below we consider the errors in determination of principal π_{im}, associated with the rotation of optical indicatrix.

2.2. Influence of indicatrix rotations on the accuracy of principal POCs

Let us choose, for example, a monoclinic crystal having three principal turning coefficients π_{5m} (π_{51}, π_{52}, π_{53}). We wish to prove that these coefficients affect a piezo-optical change in the principal refractive indices δn_i (7). The coefficients π_{5m} give rise to the off-diagonal terms δB_5 in the initially diagonal polarization constant tensor B_i . According to (3), this yields in

$$\delta B_5 = \pi_{5m}\sigma_m. \quad (20)$$

The perturbed tensor B_i then becomes

$$B'_i = \begin{pmatrix} B_1 & 0 & \delta B_5 \\ 0 & B_2 & 0 \\ \delta B_5 & 0 & B_3 \end{pmatrix}. \quad (21)$$

This is why the equation (10) for the perturbed indicatrix will take the form

$$B_1 x_1^2 + B_2 x_2^2 + B_3 x_3^2 + 2\delta B_5 x_1 x_3 = 1. \quad (22)$$

To determine the new half-axes of the indicatrix, we should reduce formula (22) to the canonical form,

$$\lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_3 x_3^2 = 1, \quad (23)$$

where λ_1, λ_2, λ_3 are the eigenvalues of the perturbed tensor. These λ_i determine the new values of the indicatrix half-axes, and so the new refractive indices n'_i. Determination of λ_i consists in solving the characteristic (or secular) equation [9,10]:

$$\begin{vmatrix} B_1 - \lambda & 0 & \delta B_5 \\ 0 & B_2 - \lambda & 0 \\ \delta B_5 & 0 & B_3 - \lambda \end{vmatrix} = 0. \quad (24)$$

Calculating the determinant leads to the equation

$$(B_2 - \lambda)\left[(B_1 - \lambda)(B_3 - \lambda) - \delta B_5^2\right] = 0,$$

with its obvious solutions given by

$$\lambda_2 = B_2;$$
$$\lambda_{1,3} = \frac{B_1 + B_3}{2} \pm \sqrt{\frac{(B_1 - B_3)^2}{4} + \delta B_5^2}. \quad (25)$$

Here the sign "+" corresponds to λ_1 and "–" to λ_3.

We conclude from (25) that the new refractive index n'_2 and the old one n_2 are identical, because $\lambda_2 = B_2 = 1/n_2^2$, while n_1 and n_3 change by the values

$$\delta n'_{1,3} = n'_{1,3} - n_{1,3} = \frac{1}{\sqrt{\lambda_{1,3}}} - \frac{1}{\sqrt{B_{1,3}}} = \\ = \mp\frac{\delta B_5^2}{2B_{1,3}^{3/2}(B_1 - B_3)} = \mp\frac{\pi_{5m}^2\sigma_m^2}{2B_{1,3}^{3/2}(B_1 - B_3)}, \quad (26)$$

where the sign "–" corresponds to $\delta n'_1$, and "+" to $\delta n'_3$. We are to emphasize that the equation (26) has been obtained with neglecting the small value δB_5^2 in the denominator, for a smallness of δB_i (i = 4, 5, 6) represents an empiric postulate valid for the most of crystals:

$$\delta B_i << B_i, (i = 1,2,3);$$
$$\delta B_i << B_i - B_j, (i, j = 1,2,3). \quad (27)$$

Hence, the relation (26) proves that the turning POCs do not impose rotation of the indicatrix only, but also change the principal refractive index n_i (i = 1,2,3). The comparison of δn_i and $\delta n'_i$ is given below (see formulae (33) and (34)).

Now we clarify a link between the coefficients π_{5m} and the angles of the indicatrix rotation. For this aim, we should find the coordinates x_i of the eigenvectors $\vec{\lambda_i}$ of tensor B'_i from (21) in the principal coordinate system X_1, X_2, X_3. According to (21), these coordinates x_1, x_2, x_3 satisfy the equation system [11–13]

$$\begin{aligned}&(B_1-\lambda)x_1+\delta B_5 x_3=0,\\&(B_2-\lambda)x_2=0,\\&\delta B_5 x_1+(B_3-\lambda)x_3=0.\end{aligned} \qquad (28)$$

Inserting λ_1 into (28), we find the coordinates x_1, x_2 and x_3 of the eigenvector $\vec{\lambda_i}$:

$$\begin{aligned}&x_2=0,\quad x_3=-\frac{B_1-\lambda_1}{\delta B_5}x_1\\&x_3=-\frac{\delta B_5}{B_3-\lambda_1}x_1\end{aligned} \qquad (29)$$

The angle between the eigenvector $\vec{\lambda_1}$ ($\vec{\lambda_1} \parallel X'_1$) and the principal axis X_1 (i.e., the rotation angle of the indicatrix) is $\tan\alpha = x_3/x_1$ (fig. 3). Inserting the expression for λ_1 from (26) into the second or third equation (29) and neglecting the small values δB_5^2 in the denominator, we find:

$$\tan\alpha=\frac{\delta B_5}{B_1-B_3}. \qquad (30)$$

The same way, inserting λ_3 into (28) gives

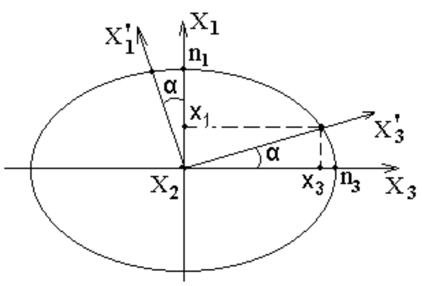


Fig.3. Section of perturbed indicatrix by the plane perpendicular to X_2.

the coordinates of the eigenvector $\vec{\lambda_3}$ and the rotation angle $\vec{\lambda_3}$ ($\vec{\lambda_3} \parallel X'_3$) we get $\tan\alpha = -x_1/x_3$ (see fig. 3), the result that coincides with (30).

Finally, taking formula (25) and the relations $\lambda=\lambda_2$ and $\lambda_2 = B_2$ into account, we obtain from (28) the coordinates of the eigenvector $\vec{\lambda_2}$: x_1=0, $x_2 \neq 0$, x_3=0. The vector $\vec{\lambda_2}$ is directed along the principal axis X_2 and so this axis does not rotate.

Inserting the values of δB_5 and the turning coefficients π_{5m} from (20) into (30), we write the relations for finding the turning POCs from the measured angle α of the indicatrix rotation around X_2 under the action of σ_m:

$$\pi_{5m}=\frac{(B_1-B_3)\tan\alpha}{\sigma_m}. \qquad (31)$$

We also write, without proving, the analogous relations for the turning coefficients π_{4m} and π_{6m}. They include the indicatrix rotations around the axes X_1 and X_3, respectively, under the action of σ_m (m=1, 2, 3):

$$\begin{aligned}&\pi_{4m}=\frac{(B_3-B_2)\tan\alpha}{\sigma_m}\\&\pi_{6m}=\frac{(B_2-B_1)\tan\alpha}{\sigma_m}.\end{aligned} \qquad (32)$$

The diagonal turning–shift coefficients π_{44}, π_{55}, and π_{66} can be also determined on the basis of the known rotations of the indicatrix around the axes X_1, X_2 and X_3, respectively. We shall consider this question below (see formulae (52–55)).

Now we compare the contributions of $\delta n_{1,3}$ and $\delta n'_{1,3}$. For this aim, we insert formulae (30) or (31) into (26) and obtain

$$\begin{aligned}\delta n'_{1,3}&=\mp\frac{(B_1-B_3)\tan^2\alpha}{2B_{1,3}^{3/2}}\approx\\&\approx\pm\Delta n_2\cdot\tan^2\alpha,\end{aligned} \qquad (33)$$

where $\Delta n_2 = n_3 - n_1$ is the optical birefringence. After using the typical values Δn_k = 0.1 – 0.001,

$\pi_{im} = 3\cdot10^{-12}$ m^2/N and $\sigma_m = 10^7$ N/m^2 = =100 kG/cm^2 in (33) and (16) (with such the σ_m values, we have $\alpha \le 0{,}2$ deg), we obtain

$$\delta n'_i = (10^{-2} \div 10^{-4})\delta n_i . \qquad (34)$$

This is correct for the most of known crystals.

One can see that the contribution of the π_{5m} coefficients to the change of the principal refractive indices n_1 and n_3 is by 2–4 orders of magnitude less than that of the principal POCs. As a result, the neglect of this contribution involves the error in the principal coefficients π_{im} which is, as a rule, considerably less than 1%. Since the accuracy for the absolute POCs, achievable with the modern methods, does not exceed 10%, the neglect of $\delta n'_i$ in the determination of principal π_{im} is absolutely justified. Besides, it is demonstrated in [14] that the indicatrix rotation by the angle α brings in most cases (e.g., within the interferometric method) the experimental error $\delta n^*_{1,3}$, which is given by the expression similar to (26), though with the opposite sign. That is why we have

$$\delta n'_{1,3} + \delta n^*_{1,3} = 0. \qquad (35)$$

Thus, the contribution of $\delta n'_{1,3}$ into the change of the principal refractive indices caused by the symmetry is compensated by the experimental error $\delta n^*_{1,3}$. Basing on (34) and (35), we could therefore state that the principal POCs π_{im} for all the symmetry groups may be determined without "looking back" on the turning POCs.

2.3. Determination of shift POCs

Below we give a particular example for the monoclinic crystals having three such coefficients (π_{15}, π_{25} and π_{35}). For their determination, it is necessary to apply a pressure such that the component σ_5 of the stress tensor be non-zero. A 45°-cut sample shown in fig. 4 seems to be optimum in this relation. Then the unity vector $\vec{p}\,(a, b, c)$ of the uniaxial pressure along the direction 5 has the directional cosines $a = c = \cos 45° = \sqrt{2}/2$ and $b = \cos 90° = 0$, and the tensor σ_m, according to (7), is as follows:

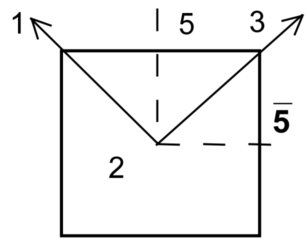


Fig.4. Scheme of sample orientation used for determination of coefficients π_{i5} and π_{5m}.

$$\sigma_m = [\sigma/2,\ 0,\ \sigma/2,\ 0,\ \sigma/2,\ 0]. \qquad (36)$$

One can see that only the components σ_1, σ_3 and σ_5 are non-zero. Using formulae (3) or (5) and taking into account that $\pi_{im} = 0$ for some coefficients, we obtain the changes in the optical polarization constants $\delta B_1, \ldots, \delta B_6$:

$$\begin{aligned}
&\delta B_1 = \frac{\sigma}{2}(\pi_{11} + \pi_{13} + \pi_{15}),\ \delta B_4 = 0,\\
&\delta B_2 = \frac{\sigma}{2}(\pi_{21} + \pi_{23} + \pi_{25}),\\
&\delta B_5 = \frac{\sigma}{2}(\pi_{51} + \pi_{53} + \pi_{55}),\\
&\delta B_3 = \frac{\sigma}{2}(\pi_{31} + \pi_{33} + \pi_{35}),\ \delta B_6 = 0 .
\end{aligned} \qquad (37)$$

Inserting (37) into (10), the following equation of perturbed indicatrix might be derived:

$$\begin{aligned}
&\left[B_1 + \frac{\sigma}{2}(\pi_{11} + \pi_{13} + \pi_{15})\right]x_1^2 +\\
&+\left[B_2 + \frac{\sigma}{2}(\pi_{21} + \pi_{23} + \pi_{25})\right]x_2^2 +\\
&+\left[B_3 + \frac{\sigma}{2}(\pi_{31} + \pi_{33} + \pi_{35})\right]x_3^2 =\\
&= \sigma(\pi_{51} + \pi_{53} + \pi_{55})x_1 x_3 = 1.
\end{aligned} \qquad (38)$$

Let the light propagate along X_2 ($k = 2$). Then the polarization is possible along X_1 and X_3 ($i = 1, 3$ – see fig. 4). If $i = 1$, the intersection of the indicatrix (38) and the axis X_1 gives the coordinate x_1 that corresponds to the new refractive index along X_1 (n'_1). It is clear that the coordinates $x_2 = x_3 = 0$. Keeping this condition and formula (37) in mind, we obtain for $x_1 \equiv n'_1$

$$\left[B_1 + \frac{\sigma}{2}(\pi_{11} + \pi_{13} + \pi_{15})\right]x_1^2 = 1, \qquad (39)$$

or

$$n'_1 = x_1 = 1/\sqrt{B_1 + (\pi_{11} + \pi_{13} + \pi_{15})\sigma/2}.$$

Since $n_1 = 1/\sqrt{B_1}$ for the unperturbed indicatrix, the change in n_1 caused by the stress σ along the direction 5 (m=5) becomes

$$\delta n_1 = n'_1 - n_1 = = -(\pi_{11} + \pi_{13} + \pi_{15})\sigma / 4B_1^{3/2} = = -\frac{\sigma}{4}(\pi_{11} + \pi_{13} + \pi_{15})n_1^3. \qquad (40)$$

Quite similar, for the same direction of light propagation (k=2) and the other polarization i = 3 (fig. 4) ($x_2 = x_1 = 0$) we obtain from (38)

$$\left[B_3 + \frac{\sigma}{2}(\pi_{31} + \pi_{33} + \pi_{35})\right]x_3^2 = 1,$$

$$n'_3 = x_3 = 1/\sqrt{B_3 + (\pi_{31} + \pi_{33} + \pi_{35})\sigma/2},$$

whence, on taking the relation $n_3 = 1/\sqrt{B_3}$ into account, one finally has

$$\delta n_3 = n'_3 - n_3 = = -\frac{\sigma}{4}(\pi_{31} + \pi_{33} + \pi_{35})n_1^3. \qquad (41)$$

Let us now reverse the pressure direction (from m=5 to $m=\bar{5}$ – see fig.4). Then $\vec{p} = \vec{p}\,(\sqrt{2}/2, 0, -\sqrt{2}/2)$, and the tensor σ_m, in accordance with (7), looks as σ_m = (σ/2, 0, σ/2, 0, –σ/2, 0). This differs from the result (36) by the sign of the component σ_5. Therefore, the signs near the coefficients π_{15}, π_{25}, π_{35} and π_{55} in (37) and (38) change from "+" to "–", and we obtain the additional relation for both π_{15} and π_{35}, which is needed for determining each of them. They differ from (40) and (41) by the signs near π_{15} and π_{35} only:

$$\delta n_1 = -\frac{\sigma}{4}(\pi_{11} + \pi_{13} - \pi_{15})n_1^3;$$
$$\delta n_3 = -\frac{\sigma}{4}(\pi_{31} + \pi_{33} - \pi_{35})n_1^3. \qquad (42)$$

Determination of π_{25}. One can see from the equation (38) and fig. 4 that the determination of the shift coefficient π_{25} should imply the polarization i = 2. It is possible for the light direction k=5 or $k=\bar{5}$. Let it be $k=\bar{5}$. Then we have m=5 (the case of $m=\bar{5}$ is also possible, but it is difficult to provide the common directions of the pressure and the light propagation). The equation of the perturbed indicatrix for m=5 is already ascertained (see formula (38)). To find δn_2, we intersect the surface (38) by the axis X_2. Then $x_1 = x_3 = 0$ and we get from (38)

$$[B_2 + (\pi_{21} + \pi_{23} + \pi_{25})\sigma/2]x_2^2 = 1,$$

$$n'_2 = x_2 = 1/\sqrt{B_2 + (\pi_{21} + \pi_{23} + \pi_{25})\sigma/2},$$

whence, on taking $n_2 = 1/\sqrt{B_2}$ into account,

$$\delta n_2 = n'_2 - n_2 = = -\frac{\sigma}{4}(\pi_{21} + \pi_{23} + \pi_{25})n_2^3. \qquad (43)$$

Changing the directions of light propagation and the pressure (k=5, $m=\bar{5}$), we have another equation for π_{25}:

$$\delta n_2 = -(\pi_{21} + \pi_{23} - \pi_{25})\sigma \cdot n_2^3. \qquad (44)$$

2.4. Determination of turning POCs on the basis of refractive indices

Let us give a single example for monoclinic crystals. We consider at first the case of the π_{52} coefficient. Here the experimental conditions are as follows: m = 2, $k = \bar{5}$ and i = 5 (for the same sample as in chapter 2.3 – see fig. 4). According to (3) or (5), we have $\delta B_1 = \pi_{12} \times \sigma$; $\delta B_2 = \pi_{22} \times \sigma$; $\delta B_3 = \pi_{32} \times \sigma$; $\delta B_4 = \delta B_6 = = 0$; and $\delta B_5 = \pi_{52} \cdot \sigma$. Therefore, the perturbed indicatrix (10) takes the form:

$$(B_1 + \pi_{12}\sigma)x_1^2 + (B_2 + \pi_{22}\sigma)x_2^2 + + (B_3 + \pi_{32}\sigma)x_3^2 + 2\pi_{52}x_1x_3 = 1, \qquad (45)$$

The condition $x_1 = x_3$; $x_2 = 0$ is correct for the polarization i=5 (fig. 5). Inserting the latter into (45), we arrive at

$$(B_1 + B_3 + \pi_{12}\sigma + \pi_{22}\sigma + 2\pi_{52}\sigma)x_1^2 = 1,$$
$$x_1 = 1/\sqrt{B_1 + B_3 + (\pi_{12} + \pi_{22} + 2\pi_{52})\sigma}, \qquad (46)$$

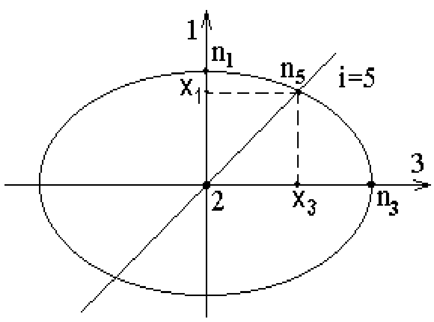


Fig.5. Intersection of the indicatrix by the line $x_1 = x_3$, $x_2 = 0$ ($i = 5$).

One can see from fig.5 that $n'_5 = \sqrt{x_1^2 + x_3^2} = x_1\sqrt{2}$, or

$$n'_5 = \sqrt{2} / \sqrt{B_1 + B_3 + (\pi_{12} + \pi_{22} + 2\pi_{52})\sigma}\,.$$

Making use of the latter relation at σ=0, we obtain $n_5 = \sqrt{2} / \sqrt{B_1 + B_3}$ for the unperturbed indicatrix, while the piezo-optical change in the refractive index n_5 is given by

$$\delta n_5 = n'_5 - n_5 =$$

$$= -(\pi_{12}+\pi_{22}+2\pi_{52})\cdot\sigma/[\sqrt{2}\,(B_1+B_3)^{3/2}]. \quad (47)$$

Under the "symmetric" experimental conditions m=2, k=5 and $i=\bar{5}$, the condition for intersecting the indicatrix (45) and the straight line, coinciding with $i=\bar{5}$, may be written as $x_1 = -x_3$, $x_2 = 0$. Therefore, the sign minus should now be near $2\pi_{52}$ in (46) and (47). So we derive another relation for determination of π_{52}.

Determination of turning POCs π_{51} and π_{53}. It is necessary to ensure such the experimental conditions, under which the components σ_1 and σ_3 are non-zero, whereas i=5 or $i=\bar{5}$ for the polarization. Such the conditions are as follows: (1) m=5, $k=\bar{5}$ and i=5, and the "symmetric" ones (2) $m=\bar{5}$, k=5 and $i=\bar{5}$.

Specifying the indicatrix equation (38) for the conditions (1), taking the pressure direction as m=5 (then the tensor σ_m is given by (36)) and considering the intersection of (38) by the straight line $x_1 = x_3$, $x_2 = 0$ (this corresponds to i=5), we have $n'_5 = x_1\sqrt{2}$ (σ≠0) and $n_5 = x_1\sqrt{2}$ (σ = 0) – everything like in the case of formula (47). Then the relation for δn_5 is also similar:

$$\delta n_5 = n'_5 - n_5 = \frac{\sigma\sqrt{2}}{4(B_1 + B_3)^{3/2}} \times$$

$$\times[\pi_{11} + \pi_{13} + \pi_{15} + \pi_{31} + \pi_{33} + \,. \quad (48)$$

$$+ \pi_{35} + 2(\pi_{51} + \pi_{53} + \pi_{55})]$$

For the conditions (2) (see above), we obtain the analogous expression, though with the sign minus near the sum π_{5m} in the brackets and near π_{55}. Solving these two equations with respect to $\pi_{51} + \pi_{53}$ and π_{55}, we get the sum $\pi_{51} + \pi_{53}$ and π_{55} separately.

Now a question arises how π_{51} and π_{53} can be separated? For this aim, it is suggested in [13] to make a sample not of 45°-orientation (see fig. 4), but, e.g., of 30° (α=30°, fig. 6). Then the equation (48) contains $\pi_{51}\cos^2\alpha + \pi_{53}\sin^2\alpha$ rather than the sum $\pi_{51} + \pi_{53}$. When being combined with the previously known sum $\pi_{51} + \pi_{53}$, this should allow to determine $\pi_{51} + \pi_{53}$ separately. The relations for π_{5m} that account for the elasticity effect are given in a complete form in table 2.

It is clear that all the mentioned recommendations as for determining the turning coefficients π_{51} and π_{53} on the basis of δn_i, especially those taking the relations for the case of α≠45° into account, have rather fundamental character. Let us consider that each of the known coefficients in the expressions like (48) can be determined with the error of β≈10%. There are six such coefficients π_{im} in (48), and the errors β are additive. Then the sum $\pi_{51} + \pi_{53} + \pi_{55}$ can be determined with the mean-

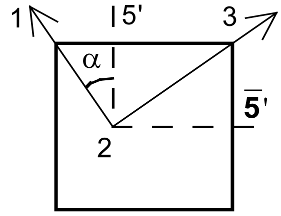


Fig.6. Scheme of sample orientation for a separate determination of POCs π_{51} and π_{53}.

square error $\beta' = \sqrt{7\beta^2} = \beta\sqrt{7}$ =26%. Moreover, another measurement must be performed for the case of "symmetric" conditions for the sum $\pi_{51}+\pi_{52}$, and one more for the sum $\pi_{51}\cos^2\alpha+\pi_{53}\sin^2\alpha$. As a consequence, the error increases additionally by $\beta\sqrt{2}$ =14%. Hence, the total error amounts about 40% or even over 50%, if the elasticity is taken into consideration. A labour–consuming character of the experiment should be also added to deficiencies of this method. Therefore we shall consider below (in chapter 2.6) the alternative possibilities to determine the turning coefficients π_{5m} (as well as the diagonal turning–shift POCs π_{44}, π_{55} and π_{66}) based on the experimental data for the pressure–induced rotations of the optical indicatrix.

2.5. Determination of turning–shift POCs π_{44}, π_{66}, π_{64} and π_{46}

In order to determine π_{44}, we are to ensure the experimental conditions m=4, $k=\bar{4}$ and i=4 (fig. 7). Then $\vec{p}=\vec{p}\,(0,\ 1/\sqrt{2},\ 1/\sqrt{2})$ and σ_m = [0, σ/2, σ/2, σ/2, 0, 0]. Writing out the equation of the perturbed indicatrix and crossing it by the straight line x_1 = 0, x_2 = x_3, which corresponds to the direction i = 4, we obtain $n_4' = x_2\sqrt{2}$ (σ≠0), $n_4 = x_2\sqrt{2}$ (at σ = 0), together with their difference:

$$\delta n_4 = -\frac{\sigma\sqrt{2}}{4(B_2+B_3)^{3/2}}\times \\ \times(\pi_{22}+\pi_{23}+\pi_{32}+\pi_{33}+2\pi_{44}). \qquad (49)$$

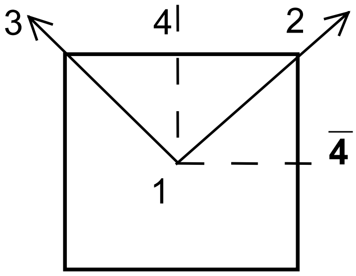


Fig.7. Scheme of sample orientation for determination of POCs π_{i4} and π_{4m} (i, m= 1,2,...,6).

The expression for $\delta n_{\bar{4}}$ under the "symmetric" conditions $m=\bar{4}$, k=4, $i=\bar{4}$ is the same as (49).

The conditions for determination of π_{66} are m=6, $k=\bar{6}$, i=6 or $m=\bar{6}$, k=6, $i=\bar{6}$ (fig. 1), in which case we also arrive at the relation:

$$\delta n_6 = \delta n_{\bar{6}} = -\frac{\sigma\sqrt{2}}{(B_1+B_2)^{3/2}}\times \\ \times(\pi_{11}+\pi_{12}+\pi_{21}+\pi_{22}+2\pi_{66}). \qquad (50)$$

Determination of the turning–shift POCs π_{46} and π_{64} represent the most complicated task. For example, in case of π_{64} the index for the polarization should be i=6 (the diagonal between X_1, X_2) and the component σ_4 of the tensor σ_m should be simultaneously non-zero. This can be done with a sample orientation shown in fig. 8 (see the proof in [13]).

Then the vector $\vec{p}$ and the tensor σ_m have all their components non-zero under the condition of $m \perp B$ ($\vec{p}$ being directed along the diagonal of 3 and 6): $\vec{p}=\vec{p}\,$(cos60°, cos60°,

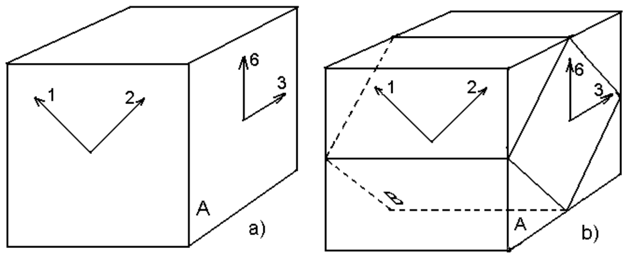


Fig.8. Scheme of sample orientation necessary for determination of the POC π_{64}: (a) original sample with 45°–orientation with respect to the axes X_1 and X_2; (b) additionally made 45°–cut with respect to the axis X_3 and the direction 6.

cos45°) and σ_m=[1/4, 1/4, 1/2, $\sqrt{2}$/4, $\sqrt{2}$/4, 1/4]σ. The light should propagate perpendicular to the side A ($k\perp A$) and the polarization i = 6 is needed. Performing the well-known operations (calculations for the perturbed indicatrix, its crossing by the straight line $x_1 = x_2$, $x_3 = 0$, and the refractive indices n'_6 and n_6) this time, we get:

$$\begin{aligned}\delta n_6 = &-\frac{\sigma}{\sqrt{2}}\Big(\frac{\pi_{11}}{4}+\frac{\pi_{12}}{4}+\frac{\pi_{13}}{2}+\\ &+\frac{\pi_{15}}{2\sqrt{2}}+\frac{\pi_{21}}{4}+\frac{\pi_{22}}{4}+\frac{\pi_{23}}{2}+\\ &+\frac{\pi_{25}}{2\sqrt{2}}+\frac{\pi_{64}}{\sqrt{2}}+\frac{\pi_{66}}{2}\Big)/(B_1+B_2)^{3/2}.\end{aligned} \quad (51)$$

To determine π_{46}, the polarization along the direction i=4 (the diagonal of X_2 and X_3) and a non-zero σ_6 component of the tensor σ_m are necessary (the scheme is analogous to that in fig.8, see also table 2.1 for the case of π_{46}). Then we have $\vec{p} = \vec{p}$ (cos45°, cos60°, cos60°) and σ_m = [1/2, 1/4, 1/4, 1/4, $\sqrt{2}$/4, $\sqrt{2}$/4]σ. Now it is easy to write the equation of the perturbed indicatrix and cross it by the straight line $x_1 = 0$, $x_2 = x_3$, that corresponds to the condition i=4. The expression for δn_4 appears to be analogous to (51), though it now includes π_{46} (see table 2.1). One can see from (51) that the accuracy of π_{64} determination (the same refers to π_{46}) is equal to $\beta'=\beta\sqrt{10}\approx 35\%$, or above 50% with taking the elasticity into account. Another way for determining these POCs is pointed to in chapter 2.6.

2.6. Determination of some POCs from the induced rotations of the indicatrix

Determination of π_{51} and π_{53}. If the stresses $\sigma_1 = \sigma$ or $\sigma_3 = \sigma$ are acting on a sample of monoclinic crystal with its sides perpendicular to the axes X_1, X_2 and X_3, , we obtain from (3) $\delta B_5 = \pi_{51}\sigma$ or $\delta B_5 = \pi_{53}\sigma$. Inserting these expressions into (30), we have

$$\begin{aligned}\pi_{51} &= (B_1 - B_3)\tan\alpha_2/\sigma_1,\\ \pi_{53} &= (B_1 - B_3)\tan\alpha_2/\sigma_3,\end{aligned} \quad (52)$$

where the index 2 of the angle α_2 means the rotation about the axis X_2.

We do not represent the analogous expression for π_{52}, because it is hard to realize the experimental geometry for which $k=m=2$. When the stress is applied along the direction m=5, the tensor σ_m has three components (36) $\sigma_1 = \sigma_3 = \sigma_5 = \sigma/2$ but not one. So the expression for π_{55} is somewhat complicated. The quantity δB_5 is formed by the sum of coefficients $\pi_{51} + \pi_{53} + \pi_{55}$ (see formula (37)). Inserting (37) into (30), we get:

$$\pi_{55} = \frac{2}{\sigma}(B_1 - B_3)\tan\alpha_2 - (\pi_{51} + \pi_{53}). \quad (53)$$

Thus, π_{55} may be determined on the basis of α_2, whenever a preliminarily information on π_{51} and π_{53} is available.

The relations applied for determination of π_{44} and π_{66} from the rotations α_1 and α_3 (around X_1 and X_3, respectively) are more simple, because the turning POCs π_{4m} and π_{6m} are equal to zero for monoclinic crystals:

$$\begin{aligned}\pi_{44} &= \frac{2}{\sigma}(B_3 - B_2)\tan\alpha_1;\\ \pi_{66} &= \frac{2}{\sigma}(B_2 - B_1)\tan\alpha_3.\end{aligned} \quad (54)$$

Let us add the following notice to formula (54). When determining π_{44}, we apply the pressure σ along the direction m=4 and so we have σ_m = [0, σ/2, σ/2, σ/2, 0, 0]. Here the components $\sigma_2 = \sigma_3 = \sigma/2$ are non-zero. They induce the additional indicatrix rotation around the axis X_2, because both π_{51} and π_{53} are non-zero. Such the rotation changes only the refractive index of the light wave polarized in the plane X_1, X_3 (k=1), and the polarization plane does not therefore change. In other words, the rotation around X_1 is related to the coefficient π_{44} only. The same remark also concerns the determination of π_{66} from the indicatrix rotation around X_3.

Similarly to the case of derivation of (53) for monoclinic crystals, it is easy to show that the expression (53) is equally correct for triclinic crystals. As for the coefficients π_{44} and π_{66}, the following formulae are correct:

$$\begin{aligned}\pi_{44} &= \frac{2}{\sigma}(B_3 - B_2)\tan\alpha_1 - (\pi_{43} + \pi_{42});\\ \pi_{66} &= \frac{2}{\sigma}(B_2 - B_1)\tan\alpha_3 - (\pi_{62} + \pi_{61}),\end{aligned} \qquad (55)$$

because $\delta B_4 = (\pi_{42} + \pi_{43} + \pi_{44})\sigma/2$ for σ acting along m=4 and $\delta B_6 = (\pi_{61} + \pi_{62} + \pi_{66})\sigma/2$ for m=6. Inserting these expressions into (30) with the appropriate cyclic change of indices near $(B_1 - B_3)$, we have again the relation (55).

The relations (54) are correct for rhombic crystals, since the turning POCs, entering (55), are equal to zero, and the relations (53) with $\pi_{51} = \pi_{53} = 0$ are valid.

Regarding optically uniaxial crystals, except for trigonal ones, we may recommend to use the expression (54) only for $\pi_{44} \equiv \pi_{55}$ (since $\pi_{4m} = 0$), when the light propagates perpendicular to the optical axis. The expression for π_{66} (k=3) in this case lacks its meaning, because it has been earlier obtained from (30) under the condition $\delta B_6 \ll B_2 - B_1$ (see formula (27)). On the basis of (30) and the latter condition, we gain the analogous condition for tanα: $\tan\alpha \ll B_2 - B_1$. However, $B_2 = B_1$ for any uniaxial crystals, and so the condition (27) is not fulfilled. As a result, formula (30) and, accordingly, formula (54) cannot be correct for the coefficient π_{66}.

Turning now to the case of trigonal uniaxial crystals, we remark that the expression (55) is correct for $\pi_{44} = \pi_{55}$, since $\pi_{43} = 0$ for such the crystals.

Determination of π_{64} and π_{46}. If the stress σ along m=6 is applied to a sample (fig. 1), we have $\sigma_m = [\sigma/2, \sigma/2, 0, 0, 0, \sigma/2]$. Let us assume that the light propagates along the direction $k=\bar{6}$. Then we can measure (e.g., with the known method of extinction) the projection α_p of rotation angle of the axis 3 and the same for the direction i=6 on the plane perpendicular to $k=\bar{6}$. This rotation would be always available because of a character of total indicatrix rotation around the axes X_1 and X_2, since the turning perturbations δB_4 and δB_5 in this case are non-zero. It is easy to see that δa_4 includes π_{46}, and δB_5 has the contributions of the coefficients π_{51} and π_{52}. Having measured the rotation α_p and the known values of π_{51} and π_{52}, we are able to find π_{46}. The details referring to determination of π_{46} and π_{64} are mentioned in the study [15].

Proceeding from the information contained in the works [16, 17], where a high accuracy for measuring the indicatrix rotation angles is declared (α=0.05–5.0 *deg*, with σ_m=100 kG/cm^2 for the most of crystals), we can affirm that the method of indicatrix rotation can, in some cases, provide a higher accuracy for determination of turning and turning-shift POCs than the method of measuring δn_i.

3. Peculiarities of POE: accounting for the elastic deformation and unambiguity in the choice of right crystal-physical coordinate system

3.1. Influence of elastic deformation on determination of POC with the interferometric method

We have already mentioned the relations (18), (19) for the principal POCs, that take into account the elastic deformations e_k of a sample in the light propagation direction. In this chapter we take into account the elastic deformation of sample for the more complicated experimental conditions. Namely, let us determine the POCs, which are not principal ones. We give the relevant examples for monoclinic crystals.

Let us take account of the elasticity (i.e., determine the expression for e_k) at the following conditions: m=2, $k=\bar{5}$, i=5 (fig. 4). We shall use the equation of a characteristic surface tensor e_k:

$$\begin{aligned}&e_1x_1^2 + e_2x_2^2 + e_3x_3^2 + e_4x_2x_3 +\\ &+ e_5x_1x_3 + e_6x_1x_2 = 1,\end{aligned} \qquad (56)$$

where e_1, e_2 and e_3 denote the mean changes of the half-axes of the characteristic surface in the coordinate system X_1, X_2, X_3, respectively, and

e_4, e_5 and e_6 the rotations of this surface around the mentioned axes.

For m=2, the tensor σ_m has the component $\sigma_2 = \sigma$ only. Using the Hook's law (17) and the matrix of elastic compliance coefficients S_{km} (k being the direction of deformation, which coincides with the light propagation direction), we can write the following expression for the tensor components e_k:

$$e_1 = S_{12}\sigma, e_2 = S_{22}\sigma, e_3 = S_{32}\sigma,$$
$$e_4 = 0, e_5 = S_{52}\sigma, e_6 = 0.$$

On this basis we rewrite (56) as:

$$\begin{aligned} & S_{12}\sigma x_1^2 + S_{22}\sigma x_2^2 + \\ & + S_{32}\sigma x_3^2 + S_{52}\sigma x_1 x_3 = 1. \end{aligned} \quad (57)$$

Using the equation of straight line $x_2 = 0$, $x_1 = -x_3$, ($k=\bar{5}$ – see fig. 9), along which we study the deformation ($k=\bar{5}$), we obtain the coordinate of the intersection point x_1:

$$\begin{aligned} & S_{12}\sigma x_1^2 + S_{32}\sigma x_1^2 - S_{52}\sigma x_1^2 = 1; \\ & x_1 = \frac{1}{\sqrt{(S_{12} + S_{32} - S_{52})\sigma}} \end{aligned} . \quad (58)$$

One can see from fig. 9 that the half-axis of the characteristic surface $1/\sqrt{e_{\bar{5}}}$ is equal to

$$\begin{aligned} & 1/\sqrt{e_{\bar{5}}} = \sqrt{x_1^2 + x_3^2} = x_1\sqrt{2} = \\ & = \sqrt{2}/\sqrt{(S_{12} + S_{32} - S_{52})\sigma} \end{aligned} .$$

The expression for $e_{\bar{5}}$ is therefore as follows:

$$e_{\bar{5}} = \frac{\sigma}{2}(S_{12} + S_{32} - S_{52}). \quad (59)$$

Inserting (59) for $e_{\bar{5}}$, along with the expression (47) for δn_5 and the expression $n_5 = \sqrt{2}/\sqrt{B_1 + B_3}$ (i=5) into equation (14), instead of n_i, we finally have

$$\begin{aligned} \delta\Delta_{\bar{5}} = & -\frac{\pi_{12} + \pi_{22} + 2\pi_{52}}{\sqrt{2}(B_1 + B_3)^{3/2}} \times \sigma d_{\bar{5}} \\ & + \frac{1}{2}(S_{12} + S_{32} - S_{52}) \times \\ & \times \sigma d_{\bar{5}}\left(\frac{\sqrt{2}}{\sqrt{B_1 + B_3}} - 1\right) \end{aligned} \quad (60)$$

This allows determining π_{52} on the basis of the measured $\delta\Delta_{\bar{5}}$ and the account for the elasticity. The relation (59) is also correct for the conditions of determination of π_{15} and π_{35} (see (42)).

At the "symmetric" conditions for determination of π_{52} (m=2, k=5, $i=\bar{5}$), we can employ the method mentioned above and obtain the following expression for e_k:

$$e_5 = \frac{\sigma}{2}(S_{12} + S_{32} + S_{52}) . \quad (61)$$

It is also correct for the "symmetric" conditions in case of the POC π_{15} and π_{35}. The complete expressions for π_{15}, π_{35} and π_{52} that account for the elasticity at the direct and "symmetric" conditions and are based on the experimental optical path changes $\delta\Delta_k$ are given in table 2.

The conditions for determination of the coefficients π_{25} and the sum $\pi_{51}+\pi_{53}+\pi_{55}$ are as follows: m=5 and $k=\bar{5}$. Then the equation of straight lines, along which the light propagates and deformation occurs, looks as x_2=0, $x_1 = -x_3$, whereas the tensor σ_m is given by σ_m = [σ/2, 0, σ/2, 0, σ/2, 0]. Therefore, the components e_k having relation to the sample deformation along the direction $x_2 = 0$, $x_1 = -x_3$, are given by

$$e_1 = \frac{\sigma}{2}(S_{11} + S_{13} + S_{15}),$$

$$e_3 = \frac{\sigma}{2}(S_{31} + S_{33} + S_{35}),$$

$$e_5 = \frac{\sigma}{2}(S_{51} + S_{53} + S_{55}),$$

and the crossing of the surface (56) and the above straight line may be written as

$$\begin{aligned} & (S_{11} + S_{13} + S_{15} + S_{31} + S_{33} + \\ & + S_{35} - S_{51} - S_{53} - S_{55})\sigma x_1^2/2 = 1. \end{aligned} \quad (62)$$

Since $S_{km}=S_{mk}$ and the semi-axis is $1/\sqrt{e_{\bar{5}}} = x_1\sqrt{2}$ (fig. 9), we find:

$$e_{\bar{5}} = \frac{\sigma}{4}(S_{11} + S_{33} - S_{55} + 2S_{13}). \quad (63)$$

It is easy to make sure that the expression for e_5 is the same under the "symmetric" conditions for π_{25} and $\pi_{51}+\pi_{53}-\pi_{55}$ (m=5, $k=\bar{5}$). Inserting (63) for $e_{\bar{5}}$, (48) for δn_5 and (43) for δn_2 into (14), we get the expressions necessary for determining the POC π_{25}, π_{55} and $\pi_{51}+\pi_{53}$.

We do not give the other expressions for e_k, because they can be derived quite similarly to those for δn_i and e_k. The corresponding formulae necessary for determination of the turning, shift and the turning–shift POCs are brought together in table 2. They refer to all symmetry classes, except for the triclinic ones, owing to a limited volume of this article. The relations for the triclinic crystals may be found in [3,18]. For a more convenience, the schemes illustrating the necessary sample orientation and the direct and "symmetric" experimental conditions are also given in table 2.

3.2. Choice of positive signs of the right co–ordinate system axes

For the most of non-principal POCs, determination of their sign is of a primary importance, along with the absolute value [19,20]. We explain this point on the example of coefficient π_{14} for trigonal symmetry class 3m. Let us write the corresponding relations for the "symmetric" experimental conditions from table 2.4 and disregard the elasticity effect:

$$\begin{aligned}&\delta n_1 = -\frac{\sigma}{4}(\pi_{12}+\pi_{13}+\pi_{14})n_1^3\\ &for\ \ i=1,\ k=\bar{4},\quad m=4;\\ &\delta n_1 = -\frac{\sigma}{4}(\pi_{12}+\pi_{13}-\pi_{14})n_1^3\\ &for\ \ i=1,\ k=4,\quad m=\bar{4}.\end{aligned} \tag{64}$$

We note that fig. 10a and 10b differ by a choice of right crystal-physical coordinate system only. One can therefore see that it is impossible to distinguish definitely between the pressure directions m = 4 and $m=\bar{4}$. The sign of the coefficient π_{14} cannot therefore be determined. The POC π_{14} enters the expression

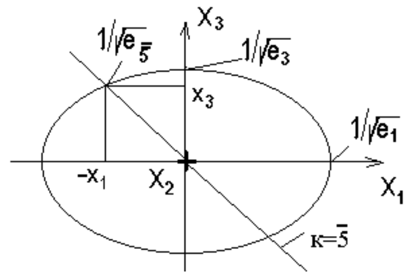


Fig.9. Crossing of characteristic surface of the deformation tensor e_k by the plane perpendicular to X_2.

for π_{44} (see table 2.4) as a known quantity. Therefore, the POC π_{44} is determined up to its absolute value.

The described uncertainty does not concern to all 32 symmetry classes, but only to those 10 classes, whose POC matrixes have turning, shift and turning-shift coefficients π_{im} entering the relations of table 2 with the signs «±» for the "symmetric" experimental conditions. 6 polar classes (1, 2, m, 3, 32 and 3m) possessing piezo-electrical effect and 4 inversion classes ($\bar{1}$, 2/m $\bar{3}$ and $\bar{3}m$) belong to these groups. The other 6 symmetry classes (4, $\bar{4}$, 4/m, 6, $\bar{6}$ and 6/m) have one only uncertainty in sign – for π_{66} (see table 2.5). For the relevant experimental geometry, π_{66} can be also determined from the birefringence or the retardation (see formulae (78)-(80)), though again up to the sign. However, there is another experimental geometry for the mentioned symmetry class, in which either π_{66} can be determined definitely (see, e.g., table 2.5) or the dependence

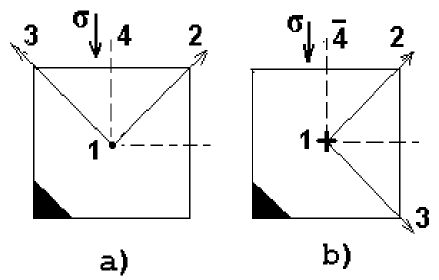


Fig.10. An example of uncertainty in the choice of right coordinate system: (a) the positive direction of the axis 1 is directed to observer, and (b) – the alternative case.

$\pi_{66} = \pi_{11} - \pi_{12}$ is correct (for the classes 6, $\bar{6}$ and 6/m). That is why the uncertainty for π_{im} for these symmetry classes is absent in practice.

The uncertainty for π_{im} in some important cases can be removed by means of an appropriate choice of the right coordinate system. It is necessary to fix the positive directions for two of the principal axes X_i only. Then the direction of the third axis would be given by the definition of right coordinate system. In case of polar symmetry classes, the positive directions of the X_i axes can be chosen on the basis of the corresponding non-zero piezo-electrical coefficients. The appropriate details are considered in [4]. For the inversion symmetry crystals, the signs of the axes X_i must be given on the basis of those of the most simple π_{im} (shift or turning ones), for which the sign uncertainty (π_{14}, π_{15}, π_{52}, π_{41}, etc.) is present.

We emphasize that it is expedient in the POE studies not to complicate recommendations for choosing the positive signs of crystallographic axes and to use the universal method of approach. Therefore, for the all 10 symmetry classes mentioned above (both polar and inversion ones) we suggest to fix the signs of X_i on the basis of POE only, namely on the basis of those π_{im}, for which the sign dualism is present. For example, the direction of the axis X_2 would be "+" if $\pi_{15} > 0$, etc. The correspon-ding recommendations are gathered in table 1.

Table 1. Recommendations for the choice of signs of the axes X_i

Symmetry classes	The recommended π_{im} to give signs of the axes X_i	Positive signs of X_i	
		Given	Deriva–tive
$1, \bar{1}, 3, \bar{3}$	$\pi_{14} > 0, \pi_{15} > 0$	X_1, X_2	X_3
$m, 2, 2/m$	$\pi_{15} > 0$	X_2	X_1, X_3
$32, 3m, \bar{3}m$	$\pi_{14} > 0$	X_1	X_2, X_3

Let us notice the following. If it is recommended there to fix the sign of one axis only, this means that we deal with non-zero POCs, whose definite determination demands choosing the sign «+» of one more axis. The same refers to the POCs which are equal, according to the POC matrix appearance, to the already chosen coefficient π_{im}.

We are at last to notice the following. If the recommended π_{im} values are small (e.g., comparable with the experimental accuracy), we should choose the other POC and further analyze, which of the axes are fixed at the chosen π_{im}.

4. Immersion method for determination of POC

The immersion–interferometric method for the studies of POE suggested in [7] allows, firstly, to remove the influence of elastic deformation on the accuracy of π_{im} and, secondly, to find this deformation and the corresponding coefficients of elastic compliance.

The essence of the method is putting a sample into the liquid with the refractive index equal to that of the crystal (n_i or $\sqrt{2/(B_i + B_j)}$) for the "diagonal" directions of polarization. Then the term (n_i–1) in (14) is equal to zero, since the unity there stands for the refractive index of the medium, in which the sample is placed. Since formula (14) is written for the sample in air, the case of sample in the immersion liquid could be described with taking into account (16) for the principal POCs:

$$\delta\Delta_k^* = \delta n_i \cdot d_k = -\frac{1}{2}\pi_{im}\sigma_m n_i^3 d_k \quad \text{or}$$
$$\pi_{im} = -\frac{\delta\Delta_k^*}{\sigma_m n_i^3 d_k}. \tag{65}$$

Similarly, the elastic contributions disappear in all the relations of table 2. For instance, we get

$$\delta\Delta_2^* = -\frac{\sigma}{4}(\pi_{31} + \pi_{33} \pm \pi_{35}) d_2 n_3^3 \tag{66}$$

for the component π_{35} of monoclinic crystals (m=5 or $\bar{5}$).

This method also allows us to determine the principal coefficients S_{im} or their sum (for the "diagonal" experimental conditions). For example, after inserting the expression for π_{im}, taken from (65), into formula (18) (it is correct for the sample in air), we obtain:

$$S_{km} = \frac{\delta\Delta_k - \delta\Delta_k^*}{d_k \sigma_m (n_i - 1)}. \qquad (67)$$

In formulae (65)-(67), $\delta\Delta_k$ and $\delta\Delta_k^*$ are the changes in the optical paths (retardation between the interferometer rays) under the action of σ_m for the sample in air and liquid, respectively.

The expressions (65) and (67) within the half-wave tension method look as

$$\pi_{im} = -\frac{\lambda}{\sigma_{im}^{(k)*} n_i^3 d_k}; \qquad S_{km} = \frac{\lambda}{2d_k (n_i - 1)} \left(\frac{1}{\sigma_{im}^{(k)}} - \frac{1}{\sigma_{im}^{(k)*}}\right) \qquad (68)$$

where $\sigma_{im}^{(k)}, \sigma_{im}^{(k)*}$ are the half-wave mechanical stresses within the air and liquid; *m*, *i* and *k* the directions of the pressure, polarization and the light propagation, respectively.

Let us notice that the refractive index of the immersion liquid n_p should not obligatory be precisely equal to n_i. Reference [21] describes the method in case of $n_p \neq n_i$. Then the expressions (65), (67) and (68) for π_{im} and S_{km} are slightly different. For example, we have

$$\pi_{im} = -\lambda \frac{\left(\frac{n_i - n_p}{\sigma_{im}^{(k)}} - \frac{n_i - 1}{\sigma_{im}^{(k)*}}\right)}{d_k n_i^3 (n_p - 1)} \qquad S_{km} = \lambda \frac{\left(\frac{1}{\sigma_{km}^{(k)}} - \frac{1}{\sigma_{km}^{(k)*}}\right)}{2d_k (n_p - 1)} \qquad (69)$$

instead of (68). It is easy to see that the expressions (69) at $n_p = n_i$ are the same as (68). The relations (69) have a wider application range: using the temperature and dispersion relations for n_p and n_i, one can study the corresponding dependences for the π_{im} coefficients.

The sign of the induced changes in the optical path ($\delta\Delta_k$ and $\delta\Delta_k^*$) or the corresponding half-wave stresses could be determined by means of inserting a glass plate, optical wedge, or Berek plate into the measuring arm of interferometer. For example, a deviation of a glass plate from the orientation perpendicular to the ray shifts the interferometric pattern, because of an increase in the optical path length. If the application of σ_m shifts the interferometric fringes in the same direction, then $\delta\Delta_k$ also increases the optical path. The sign of $\delta\Delta_k$ should therefore be «+», etc.

In this article we give only the formulae for the half-wave method in case of determination of π_{im} and S_{km}. We do not consider the other methods for determining $\delta\Delta_k$ and $\delta\Delta_k^*$, as well as the appropriate schemes and the comments to specific experimental methods, due to a restricted volume of the article.

5. POE from the birefringence and retardation, including the absolute POCs. Symmetry lowering

5.1. Piezo-optical coefficients of birefringence π^*_{km} and those of retardation π°_{km}

Let us give a strict definition of the mentioned piezo-optical coefficients. The change in the birefringence Δn_k as a result of the pressure σ_m is given by

$$\delta\Delta n_k = \delta n_i - \delta n_j, \qquad (70)$$

where δn_i, δn_j are the refractive index changes along the two mutually perpendicular polarization directions *i* and *j* in the crystal, and k is the light propagation direction, i.e., $k \perp i \perp j$.

We insert the expressions for $\delta n_{i,j}$ from (16) into (70) and so obtain

$$\delta\Delta n_k = -\frac{1}{2}(\pi_{im} n_i^3 - \pi_{jm} n_j^3)\sigma_m. \qquad (71)$$

Using the notation taken from the authoritative monograph [9],

$$\pi_{km}^* = \pi_{im} n_i^3 - \pi_{jm} n_j^3, \qquad (72)$$

one can write (71) as

$$\delta\Delta n_k = -\frac{1}{2}\pi^*_{km}\sigma_m \quad \text{or}$$
$$\pi^*_{km} = -\frac{2\delta\Delta n_k}{\sigma_m}. \qquad (73)$$

Hence, the POC π^*_{km} is the coefficient that describes the birefringence change $\delta\Delta n_k$ induced by mechanical stress σ_m.. We call it as *POC of birefringence* for that reason. From the expressions (73) and (1) we get the following relation between π^*_{km} and the Brewster's constant: $\pi^*_{km} = -2C_\sigma$. It is useful, when comparing the POCs of birefringence taken from different references.

Let us note at this point that most of accessible and reliable methods in piezo-optics are based on the analysis of light passed through a sample placed between two polarizers [1,9]. They are called as polarization-optical methods [9] and allow for finding the retardation changes $\delta\Delta_k = \delta(\Delta n_k \cdot d_k)$ induced by the stress σ_m, where Δn_k and d_k are the birefringence and sample thickness in the direction of light propagation k, respectively. Differentiating this expression, we have

$$\delta\Delta_k = \delta\Delta n_k d_k - \Delta n_k \delta d_k. \qquad (74)$$

Inserting the expression for $\delta\Delta n_k$ from (73) and the expression for $\delta d_k = e_k d_k$ into (74) and accounting the Hook's law (17), we get

$$\delta\Delta_k = -\frac{1}{2}\pi^*_{km}\sigma_m d_k + S_{km}\sigma_m d_k \Delta n_k \quad \text{or}$$
$$-\frac{2\delta\Delta_k}{\sigma_m d_k} = \pi^*_{km} - 2\Delta n_k S_{km}. \qquad (75)$$

Introducing, by analogy with (73), the notation in the l.h.s. of (75),

$$\pi^o_{km} = -\frac{2\delta\Delta_k}{\sigma_m d_k}, \qquad (76)$$

(it is this coefficient that can be determined with the polarization-optical method), we write (75) in a more compact form:

$$\pi^*_{km} = -\frac{2\delta\Delta_k}{\sigma_m d_k} + 2\Delta n_k S_{km} = \pi^o_{km} + 2\Delta n_k S_{km}. \qquad (77)$$

The physical meaning of π^o_{km} is given by (76): this is the coefficient that describes the retardation change $\delta\Delta_k$ induced by the stress σ_m. Therefore, we call the coefficient π^o_{km} as *the POC of retardation*.

The quantity $\delta\Delta_k$ in (76) is related to the thickness d_k. As a consequence, the units of the coefficients π^*_{km} and π^o_{km} are the same. As a rule, these are "brewsters" (1Br=10^{-12} m^2/N). The absolute POCs π_{im}, considered in detail in chapters 2 and 3 are just expressed in these units. For the correct account for elasticity, the coefficients S_{km} should be transformed to the units of 10^{-12} m^2/N.

In order to determine π^*_{km} on the basis of π^o_{km} or $\delta\Delta_k$ from (77), it is necessary to take correctly into account the signs of terms in the r.h.s. of (77). Let us agree to attribute the sign «+» to the natural Δn_k and the corresponding retardation $\Delta n_k \cdot d_k$. If the induced part $\delta\Delta_k$ increases the absolute value of the retardation $\Delta n_k \cdot d_k$, the sign of $\delta\Delta_k$ would be «+», and vice versa. For instance, we may place a quartz wedge, with its indicatrix axes parallel to the axes of the sample indicatrix, between the polarizers (before or after the sample). If this causes the interference extrema or the photovoltaic signal measured after the analyzer to change in the same direction, as under the action of σ_m, then we attribute the sign «+» to $\delta\Delta_k$, etc. At the same time, we ascribe the sign «–» to compressing stresses and the sign «+» to tension stresses.

Such the method for determining the $\delta\Delta_k$ sign allows to determine distinctly the absolute value of π^*_{km} coefficients. Furthermore, we wish that the coefficients π^*_{km}, determined on the basis of (77), and π^*_{km}, determined on the basis of the absolute coefficients π_{im} (from the expression (72)), correlate with each other in both the absolute value and sign. It is necessary to use for this purpose the generalized rule for the signs of $\delta\Delta_k$ formulated in [20,22,23]. This is especially urgent when π^*_{km} is determined at the complicated experimental conditions (i, k, $m > 3$). Then π^*_{km} includes the contributions of a large number of absolute POCs π_{im} (see table 2). We do not describe this rule here, instead referring the reader to the references mentioned above.

We are to emphasize that accounting for the elasticity in the process of determination of POC of birefringence π^*_{km} is important because of a large value of natural birefringence. If Δn_k=0 (isotropic bodies, cubic crystals and isotropic directions in uniaxial and biaxial crystals) or $\Delta n_k \to 0$ (for most crystals with $\Delta n_k \leq 0{,}01$), the contribution of the elastic term to π^o_{km} is equal to zero or does not exceed 2-5% [23,24] (see formula (77)). This is less than the typical accuracy of polarization-optical methods (~10% [25,26]). With large values of Δn_k (~ 0,03-0,10), the elastic term should be necessarily taken into account. Moreover, sometimes $\delta\Delta_k$ consists mainly of the elastic contribution, while π^*_{km} tends to zero. For example, the values $\delta\Delta_3$ and, correspondingly, π^o_{31} for triglucine sulphate crystals [27] in the experimental geometry k=3, m=1 include about 91-100% of the elastic contribution. Thus, the birefringence change Δn_3 and the "true" coefficient π^*_{31} are equal to zero. There also happen the cases (see, e.g., [27]) when π^o_{km} and the elastic contribution $2\Delta n_k S_{km}$ are equal in their absolute values. Then we conclude that π^*_{km} =0. However, it is in fact demonstrated in [28] that the correct account for the signs of different terms in (77) leads to the conclusion that the POC π^*_{32} exceeds approximately two times π^o_{32} and $2\Delta n_3 S_{32}$.

Sometimes it is necessary to take care of the signs of $\delta\Delta_k$ and the elastic contribution, even if $\Delta n_k \to 0$. This is because S_{km} can be large, the "true" POC π^*_{km} small, while their contributions to $\delta\Delta_k$ comparable. In other words, we should use carefully the expression (77) and the law of signs for each anisotropic crystal.

5.2. Determination of absolute POCs after piezo-optic changes of retardation or birefringence

The methods for determination of the induced retardation $\delta\Delta_k$ or the birefringence $\delta\Delta n_k$ are essentially easier than the interferometric one. But they cannot be used to determine the absolute POCs. The only exceptions are the coefficients π_{66} for uniaxial crystals and $\pi_{44} = \pi_{55} = \pi_{66}$ for cubic ones. Let us demonstrate this.

It is easy to prove with the technique for finding the eigenvalues and eigenvectors of a rank-two material tensor (see chapter 2) that the light polarization direction should be along the pressure direction or perpendicular to the latter direction and the light propagation one, whenever the light propagates along the principal axis X_3 of the uniaxial crystal or along one of the principal axes X_1, X_2 and X_3 of a cubic crystal, and the uniaxial pressure is applied perpendicular to the light ray. Then we have i=6 or $i=\bar{6}$ for the polarization under the experimental conditions k=3 and m=6 (fig. 1).

Let us write the changes in the refractive indices, which cause the birefringence $\delta\Delta n_3$ along the direction k=3, under such the conditions. We use the expressions for δn_6 and $\delta n_{\bar{6}}$ taken, e.g., from the relations for π_{66} in tetragonal crystals (tables 2.5 and 2.6):

$$\delta n_6 = -\frac{\sigma}{4}(\pi_{11} + \pi_{12} + \pi_{66})n_1^3,$$

$$\delta n_{\bar{6}} = -\frac{\sigma}{4}(\pi_{11} + \pi_{12} - \pi_{66})n_1^3,$$

We obtain the expression

$$\delta\Delta n_3 = \delta n_6 - \delta n_{\bar{6}} = -\frac{1}{2}\pi_{66}\sigma \cdot n_1^3, \qquad (78)$$

enabling us to determine π_{66} on the basis of the measured birefringence. Multiplying both sides of (78) by the thickness d_3, we arrive at the expression for determining π_{66} from the induced retardation $\delta\Delta_3$ (it is just measured in the experiment):

$$\delta\Delta_3 = d_3\delta\Delta n_3 = -\frac{1}{2}\pi_{66}\sigma d_3 n_1^3 \qquad (79)$$

or, with accounting for formula (76),

$$\pi_{66} = \pi_{66}^o / n_1^3. \qquad (80)$$

In this case the POC of birefringence π^*_{66} and the POC of retardation π_{66} are equal, because the natural birefringence $\Delta n_3 = 0$ (see (77)). The expression (80) is also correct for cubic crystals (see the problem of determination of $\pi_{66} = \pi_{55} = \pi_{44}$).

The literature in the field sometimes erroneously includes the trivial relations like $\pi_{44} = \pi_{44}^* / n_4^3$ (and the analogous ones for π_{55}, π_{66}) when the light propagates along the directions $4(\bar{4}), 5(\bar{5}), 6(\bar{6})$ – see fig. 1, 4 and 7 for the monoclinic, rhombic or uniaxial crystals. That is why we give some examples of such the experimental conditions.

One can see from the experimental conditions for the coefficients π_{14} and π_{44} and the corresponding figure (see tables 2.3 and 2.4) that the polarization is possible along i=1 and i=4, and m=4 or $k=\bar{4}$. Basing on the appropriate relations, we write the expressions for δn_1 and δn_4, which are included in the birefringence $\delta\Delta n_{\bar{4}}$:

$$\delta n_1 = -\frac{\sigma}{4}(\pi_{12} + \pi_{13} + \pi_{14}) n_1^3,$$

$$\delta n_4 = -\frac{\sigma\sqrt{2}}{4(B_1 + B_3)^{3/2}} \times \qquad (81)$$

$$\times(\pi_{12} + \pi_{13} - \pi_{14} + \pi_{31} + \pi_{33} - 2\pi_{41} + 2\pi_{44}).$$

One can see that the induced birefringence $\delta\Delta_{\bar{4}} = \delta n_1 - \delta n_4$ is a complex combination of seven POCs, including the cases when $\pi_{14} = \pi_{41} = 0$ (for monoclinic, rhombic, tetragonal and hexagonal crystals). So the POE will contain also the elastic contribution from the combination coefficients S_{km} (see the same relations in tables 2.3 and 2.4).

Even in the case of cubic crystals, the light propagation along the direction $k=4(\bar{4})$ does not allow to determine π_{44} on the basis of the measured birefringence. For example, the experimental conditions m=4 and $k=\bar{4}$ give the possible polarizations directed along i=4 or i=1. The expression for δn_4 may be taken from table 2.9 (for the case of π_{44} in cubic crystals), while δn_1 is the same as in formula (81) (here it is taken into account that π_{14}=0 for the cubic crystals). We have therefore:

$$\delta\Delta n_{\bar{4}} = \delta n_1 - \delta n_4 =$$

$$= -\frac{1}{4}(\pi_{12} + \pi_{13})\sigma n_1^3 +$$

$$+\frac{1}{8}(2\pi_{11} + \pi_{12} + \pi_{13} + 2\pi_{44})\sigma n_1^3 =$$

$$= \frac{2(\pi_{11} + \pi_{44}) - (\pi_{12} + \pi_{13})}{8}\sigma n_1^3.$$

Thus, the birefringence measurements could only yield in a combination of the absolute POCs.

5.3. Induced anisotropy (symmetry lowering) at the pressure operation

It is indicated in the monograph [1] that F.Pockels "passed over in silence" the question of principle: under external pressure applied to crystal, the symmetry of the latter breaks off. This means that the uniaxial crystal becomes biaxial (if $m \neq 3$), while a cubic crystal becomes uniaxial or biaxial, depending on the specific symmetry class and the direction of pressure. In case of induced optical biaxiality in the cubic crystals, the angle between the optical axes does not depend on the σ_m value but on the pressure direction only (see more details in [1]).

It is clear that the symmetry lowering affects the Pockels' POC matrixes, and the additional coefficients must appear there. This should complicate determination of the initial coefficients π_{im}. Let us give a qualitative estimation of this effect.

The influence of the "brought anisotropy" on the accuracy of π_{im} coefficients in the uniaxial crystals can be estimated on the basis of (33). If the natural birefringence δn_k is equal to 0.1 – 0.001, then the error of determination of π_{im} on the basis of δn_i amounts β = 0.01 – 1%

(see (34)). It is understood that the value of the induced parameter $\delta\Delta n_k$ is comparable with δn_i given by (16). Inserting the typical value π_{im} = 3×10^{-12} m^2/N (for σ_m = 10^7 N/m^2 or 100 kG/cm^2) into (16), we have the estimate $\delta n_i \sim \delta\Delta n_k ==5\times10^{-5}$. If we use (33) and (34) with Δn_k=0.01, the theoretical error $\delta n'_i$ (i.e., the error of determination of π_{im}) is β=0.1%, and the change in Δn_k by the value of 5×10^{-5} leads to the error changes due to $\delta n'_i$ ($5\times10^{-5}/\Delta n_k$) as large as $\beta=5\times10^{-4}$ %. It is far outside the capability of a real experiment. As a consequence, the components of the POC matrix in the uniaxial crystals can be calculated without accounting of the induced anisotropy.

But in its essence, the question of symmetry lowering under the external pressure has been raised quite correctly. The effect is accompanied by appearance of some new POCs, whose value depends on a pressure value, and this is called [29] as a "morphic" effect. It manifests itself strikingly for the large hydrostatic pressures, in the form of a nonlinear dependence of the POE characteristics on the pressure [30–32]. However, these questions overstep the limits of this review.

It is impossible to prove the scantiness of specific π_{im} error under the symmetry break-off in the cubic crystals on the basis of (33), since Δn_k=0 there. So the only reliability criterion in case of determination of π_{im} should be the POE results themselves, i.e., the linearity mentioned above. Any deviation from the linearity shows a presence of the "morphic" effect. The size of the non-linearity must be attributed to the error of π_{im}, which can be thus estimated in each particular experiment. In order to obtain a less "morphic" error, one has, of course, to apply less pressures to the sample.

Conclusion

In this article we have presented a full enough and consistent analysis of the methodology for determination of all the absolute piezo-optical coefficients π_{im} for the crystals of all symmetry classes, except for triclinic ones. Since the triclinic crystals are rarely subjected to the investigations and the volume of the article is rather limited, the latter does not represent a serious deficiency. We refer readers to the works [3,18] where the POE in triclinic crystals is described and the corresponding working relations are cited. Table 2 gives the relations enabling one to determine all the POCs of any crystals with monoclinic or higher symmetry, by means of interferometry methods and with accounting for the elastic deformation. The easiest way to account for the influence of elastic deformation or remove it lies in using the immersion–interferometric method described in this work. We also present the formulae for determining the absolute POCs on the basis of the known rotations of indicatrix, birefringence and retardation, induced by the uniaxial pressure. In some cases they are very useful and could provide more precise evaluation of the POC than that based on interferometry methods.

Let us notice the following important point. The ambiguity in the determination of sign and absolute value (see chapter 3) is also typical for the turning POCs, when the latter are determined from the rotations of optical indicatrix. The reason is impossibility to introduce a criterion of positive (or negative) rotation angles without fixing strictly the crystal-physical axes. When the right coordinate system is chosen unambiguously, a possibility appears to formulate a sign rule for the pressure-induced angles of the indicatrix rotation α_i, based on the criterion of identity of the signs of the turning POCs found both from δn_i and α_i.

The author is grateful to Prof. R.O. Vlokh and Dr. N.M. Demyanyshyn for valuable discussions on the formulation of subject and the account of conceptual questions of this article.

Table 2. The working relations used to determine the absolute POCs by means of one–pass interferometers (*lower sign at π_{im} and S_{km} corresponds to experiment conditions within brackets*)

Table 2.1. Monoklinic system

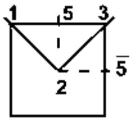
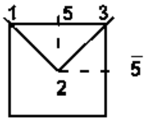
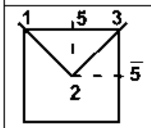
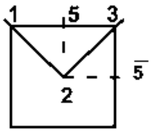
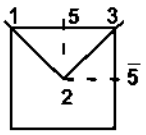
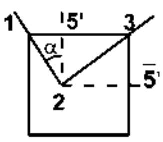
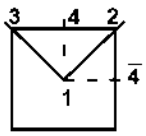
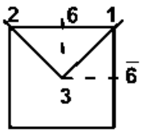
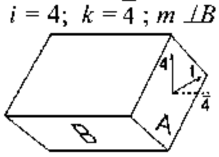
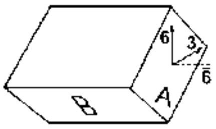
 $i = 1$, $k = 2$, $m = 5\,(\bar{5})$	π_{15}	$\delta\Delta_2 = -\frac{\pi_{11}+\pi_{13}\pm\pi_{15}}{4}\cdot\sigma\cdot d_2\cdot n_1^3 + 0.5(S_{12}+S_{23}\pm S_{25})\cdot\sigma\cdot d_2(n_1-1)$
 $i = 2$, $k = \bar{5}\,(5)$, $m = 5\,(\bar{5})$	π_{25}	$\delta\Delta_{\bar{5}(5)} = -\frac{\pi_{21}+\pi_{23}\pm\pi_{25}}{4}\cdot\sigma\cdot d_{\bar{5}(5)} n_2^3 + \frac{1}{4}(S_{11}+S_{33}-S_{55}+2S_{13})\cdot\sigma\cdot d_{\bar{5}(5)}(n_2-1)$
 $i = 3$, $k = 2$, $m = 5\,(\bar{5})$	π_{35}	$\delta\Delta_2 = -\frac{\pi_{31}+\pi_{33}\pm\pi_{35}}{4}\cdot\sigma\cdot d_2\cdot n_3^3 + 0.5(S_{12}+S_{23}\pm S_{25})\cdot\sigma\cdot d_2(n_3-1)$
 $i = 5\,(\bar{5})$, $k = \bar{5}\,(5)$, $m = 2$	π_{52}	$\delta\Delta_{5(5)} = -\frac{\pi_{12}+\pi_{32}\pm 2\pi_{52}}{\sqrt{2}(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{5(5)} + \frac{1}{2}(S_{12}+S_{23}\mp S_{25})\cdot\sigma\cdot d_{5(5)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
 $i = 5\,(\bar{5})$, $k = \bar{5}\,(5)$, $m = 5\,(\bar{5})$	π_{55}	$\delta\Delta_{\bar{5}(5)} = -\sqrt{2}\frac{\pi_{11}+\pi_{13}\pm\pi_{15}+\pi_{31}+\pi_{33}\pm\pi_{35}\pm 2(\pi_{51}+\pi_{53}\pm\pi_{55})}{4(B_1+B_3)^{3/2}}\times \sigma\cdot d_{\bar{5}(5)} + \frac{1}{4}(S_{11}+S_{33}-S_{55}+2S_{13})\cdot\sigma\cdot d_{\bar{5}(5)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
 $i = 5'$, $k = \bar{5}'$, $m = 5'$	π_{51} π_{53}	$\delta\Delta_{\bar{5}'} = -\frac{\cos\alpha\cdot d_{\bar{5}'}\cdot\sigma}{2(B_1+B_3\tan^2\alpha)^{3/2}}[\pi_{11}+(\pi_{13}+\pi_{31}+2\pi_{55})\tan^2\alpha+(\pi_{15}+2\pi_{51})\tan\alpha + (\pi_{35}+2\pi_{53})\tan^3\alpha+\pi_{33}\tan^4\alpha]+d_{\bar{5}'}\cdot\sigma\cdot\cos^4\alpha[(S_{11}+S_{33}-S_{55})\tan^2\alpha + S_{13}(1+\tan^4\alpha)+(S_{15}-S_{35})(\tan^3\alpha-\tan\alpha)](\frac{1}{\cos\alpha\sqrt{B_1+B_3\tan^2\alpha}}-1)$
 $i = 4\,(\bar{4})$, $k = \bar{4}\,(4)$, $m = 4\,(\bar{4})$	π_{44}	$\delta\Delta_{\bar{4}(4)} = -\sqrt{2}\frac{\pi_{22}+\pi_{23}+\pi_{32}+\pi_{33}+2\pi_{44}}{4(B_2+B_3)^{3/2}}\cdot\sigma\cdot d_{\bar{4}(4)} + \frac{1}{4}(S_{22}+S_{33}-S_{44}+2S_{23})\cdot\sigma\cdot d_{\bar{4}(4)}(\frac{\sqrt{2}}{\sqrt{B_2+B_3}}-1)$
 $i = 6\,(\bar{6})$, $k = \bar{6}\,(6)$, $m = 6\,(\bar{6})$	π_{66}	$\delta\Delta_{6(6)} = -\sqrt{2}\frac{\pi_{11}+\pi_{12}+\pi_{21}+\pi_{22}+2\pi_{66}}{4(B_1+B_2)^{3/2}}\cdot\sigma\cdot d_{\bar{6}(6)} + \frac{1}{4}(S_{11}+S_{22}-S_{66}+2S_{12})\cdot\sigma\cdot d_{\bar{6}(6)}(\frac{\sqrt{2}}{\sqrt{B_1+B_2}}-1)$
$i = 4$; $k = \bar{4}$; $m \perp B$ 	π_{46}	$\delta\Delta_{\bar{4}} = -\sqrt{2}\frac{\sigma\cdot d_{\bar{4}}}{8(B_2+B_3)^{3/2}}[(\pi_{22}+\pi_{23}+\pi_{32}+\pi_{33}+2(\pi_{21}+\pi_{31}+\pi_{44})+\sqrt{2}(\pi_{25}+\pi_{35} + 2\pi_{46})]+\frac{\sigma\cdot d_{\bar{4}}}{8}[S_{22}+S_{33}-S_{44}+2(S_{12}+S_{13}+S_{23})+\sqrt{2}(S_{25}+S_{35}-S_{46})]\cdot(\frac{\sqrt{2}}{\sqrt{B_2+B_3}}-1)$
$i = 6$; $k = \bar{6}$; $m \perp B$ 	π_{64}	$\delta\Delta_{\bar{6}} = \frac{-\sqrt{2}\cdot\sigma\cdot d_{\bar{6}}}{8(B_1+B_2)^{3/2}}[\pi_{11}+\pi_{12}+\pi_{21}+\pi_{22}+2(\pi_{13}+\pi_{23}+\pi_{66})+\sqrt{2}(\pi_{15}+\pi_{25}+2\pi_{64})] + \frac{\sigma\cdot d_{\bar{6}}}{8}[S_{11}+S_{22}-S_{66}+2(S_{12}+S_{13}+S_{23})+\sqrt{2}(S_{15}+S_{25}-S_{46})]\cdot(\frac{\sqrt{2}}{\sqrt{B_1+B_2}}-1)$

Table 2.2. Rhombic system

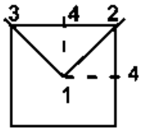
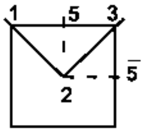
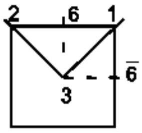
	$i = 4(\bar4)$ $k = \bar4(4)$ $m = 4(\bar4)$	π_{44}	$\delta\Delta_{\bar4(4)} = -\sqrt{2}\frac{\pi_{22}+\pi_{23}+\pi_{32}+\pi_{33}+2\pi_{44}}{4(B_2+B_3)^{3/2}}\cdot\sigma\cdot d_{\bar4(4)} + \frac{1}{4}(S_{22}+S_{33}-S_{44}+2S_{23})\cdot\sigma\cdot d_{4(4)}(\frac{\sqrt{2}}{\sqrt{B_2+B_3}}-1)$
	$i = 5(\bar5)$ $k = \bar5(5)$ $m = 5(\bar5)$	π_{55}	$\delta\Delta_{\bar5(5)} = -\sqrt{2}\frac{\pi_{11}+\pi_{13}+\pi_{31}+\pi_{33}+2\pi_{55}}{4(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{\bar5(5)} + \frac{1}{4}(S_{11}+S_{33}-S_{55}+2S_{13})\cdot\sigma\cdot d_{\bar5(5)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
	$i = 6(\bar6)$ $k = \bar6(6)$ $m = 6(\bar6)$	π_{66}	$\delta\Delta_{6(6)} = -\sqrt{2}\frac{\pi_{11}+\pi_{12}+\pi_{21}+\pi_{22}+2\pi_{66}}{4(B_1+B_2)^{3/2}}\cdot\sigma\cdot d_{\bar6(6)} + \frac{1}{4}(S_{11}+S_{22}-S_{66}+2S_{12})\cdot\sigma\cdot d_{6(6)}(\frac{\sqrt{2}}{\sqrt{B_1+B_2}}-1)$

Table 2.3. Trigonal system (classes 3 and $\bar3$)

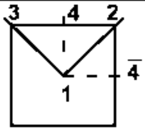
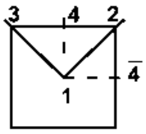
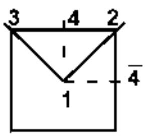
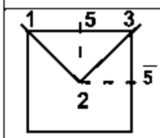
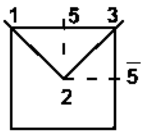
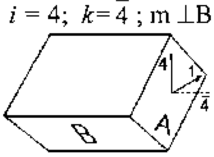
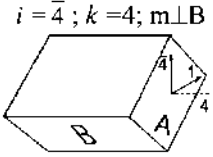
	$i = 1$ $k = \bar4(4)$ $m = 4(\bar4)$	π_{14}	$\delta\Delta_{\bar4(4)} = -\frac{\pi_{12}+\pi_{13}\pm\pi_{14}}{4}\cdot\sigma\cdot d_{\bar4(4)}\cdot n_1^3 + \frac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{23})\cdot\sigma\cdot d_{\bar4(4)}(n_1-1)$
	$i = 4(\bar4)$ $k = \bar4(4)$ $m = 1$	π_{41}	$\delta\Delta_{4(4)} = -\sqrt{2}\frac{\pi_{21}+\pi_{31}\pm 2\pi_{41}}{2(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{4(4)} + \frac{1}{2}(S_{12}+S_{13}\mp S_{14})\cdot\sigma\cdot d_{\bar4(4)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
	$i = 4(\bar4)$ $k = \bar4(4)$ $m = 4(\bar4)$	π_{44}	$\delta\Delta_{\bar4(4)} = -\sqrt{2}\frac{\pi_{11}+\pi_{13}\mp\pi_{14}+\pi_{31}+\pi_{33}\mp 2(\pi_{41}\mp\pi_{44})}{4(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{\bar4(4)} + \frac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{13})\cdot\sigma\cdot d_{\bar4(4)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
	$i = 1$ $k = 2$ $m = 5(\bar5)$	π_{15}	$\delta\Delta_2 = -\frac{\pi_{11}+\pi_{13}\pm\pi_{15}}{4}\cdot\sigma\cdot d_2\cdot n_1^3 + \frac{1}{2}(S_{12}+S_{13}\pm S_{25})\cdot\sigma\cdot d_2(n_1-1)$
	$i = 5(\bar5)$ $k = \bar5(5)$ $m = 5(\bar5)$	π_{51}	$\delta\Delta_{\bar5(5)} = -\sqrt{2}\frac{\pi_{11}+\pi_{13}\pm\pi_{15}+\pi_{31}+\pi_{33}\pm 2(\pi_{51}\pm\pi_{44})}{4(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{5(5)} + \frac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{13})\cdot\sigma\cdot d_{\bar5(5)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
$i = 4$; $k=\bar4$; $m \perp B$ 		π_{45} π_{16}	$\delta\Delta_{\bar4} = -\sqrt{2}\frac{\sigma\cdot d_{\bar4}}{8(B_1+B_3)^{3/2}}[\pi_{11}+\pi_{33}+\pi_{13}+3\pi_{31}-\pi_{14}+2(\pi_{12}+\pi_{41}+\pi_{44})-\sqrt{2}(\pi_{15}+4\pi_{51}) + \sqrt{2}(2\pi_{45}-\pi_{16})] + \frac{\sigma\cdot d_4}{8}[S_{11}+S_{33}-S_{44}+2(S_{12}+2S_{13}-S_{14})+\sqrt{2}S_{15}]\cdot(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
$i = \bar4$; $k = 4$; $m \perp B$ 		π_{45} π_{16}	$\delta\Delta_4 = -\sqrt{2}\frac{\sigma\cdot d_4}{8(B_1+B_3)^{3/2}}[\pi_{11}+\pi_{33}+\pi_{13}+3\pi_{31}+\pi_{14}+2(\pi_{12}+\pi_{41}-\pi_{44})+\sqrt{2}(4\pi_{51}-\pi_{15}) + \sqrt{2}(2\pi_{45}+\pi_{16})] + \frac{\sigma\cdot d_4}{8}[S_{11}+S_{33}-S_{44}+2(S_{12}+2S_{13}+S_{14})+\sqrt{2}S_{15}]\cdot(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$

Table 2.4. Trigonal system (classes 32, 3m and $\bar{3}$ m)

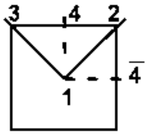
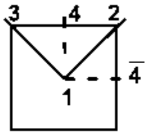
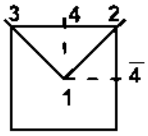
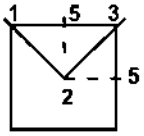
 $i = 1$ $k = \bar{4}\,(4)$ $m = 4\,(\bar{4})$	π_{14}	$\delta\Delta_{4(4)} = -\frac{\pi_{12}+\pi_{13}\pm\pi_{14}}{4}\cdot\sigma\cdot d_{4(4)}\cdot n_1^3 +$ $+\frac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{13})\cdot\sigma\cdot d_{\bar{4}(4)}(n_1-1)$
 $i = 4\,(\bar{4})$ $k = \bar{4}\,(4)$ $m = 1$	π_{41}	$\delta\Delta_{\bar{4}(4)} = -\sqrt{2}\,\frac{\pi_{21}+\pi_{31}\pm 2\pi_{41}}{2(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{\bar{4}(4)} +$ $+\frac{1}{2}(S_{12}+S_{13}\mp S_{14})\cdot\sigma\cdot d_{\bar{4}(4)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
 $i = 4\,(\bar{4})$ $k = \bar{4}\,(4)$ $m = 4\,(\bar{4})$	π_{44}	$\delta\Delta_{4(4)} = -\sqrt{2}\,\frac{\pi_{11}+\pi_{13}\mp\pi_{14}+\pi_{31}+\pi_{33}\mp 2(\pi_{41}\mp\pi_{44})}{4(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{4(4)} +$ $+\frac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{13})\cdot\sigma\cdot d_{\bar{4}(4)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
 $i = 5\,(\bar{5})$ $k = \bar{5}\,(5)$ $m = 5\,(\bar{5})$	$\pi_{55}\equiv$ $\equiv\pi_{44}$	$\delta\Delta_{\bar{5}(5)} = -\sqrt{2}\,\frac{\pi_{11}+\pi_{13}+\pi_{31}+\pi_{33}+\pi_{55}}{4(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{\bar{5}(5)} +$ $+\frac{1}{4}(S_{11}+S_{33}-S_{55}+2S_{13})\cdot\sigma\cdot d_{\bar{5}(5)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$

Table 2.5. Tetragonal system (classes 4, $\bar{4}$ and 4/m)

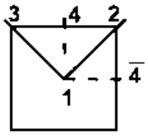
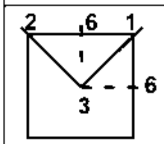
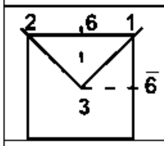
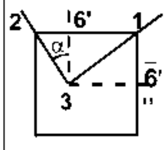
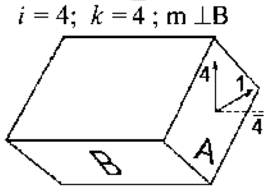
 $i = 4\,(\bar{4})$ $k = \bar{4}\,(4)$ $m = 4\,(\bar{4})$	π_{44}	$\delta\Delta_{\bar{4}(4)} = -\sqrt{2}\,\frac{\pi_{11}+\pi_{13}+\pi_{31}+\pi_{33}+2\pi_{44}}{4(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{\bar{4}(4)} +$ $+\frac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{13})\cdot\sigma\cdot d_{\bar{4}(4)}(\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$
 $i = 6\,(\bar{6})$ $k = \bar{6}\,(6)$ $m = 6\,(\bar{6})$	π_{66}	$\delta\Delta_{\bar{6}(6)} = -\frac{\pi_{11}+\pi_{12}+\pi_{66}}{4}\cdot\sigma\cdot d_{\bar{6}(6)}\cdot n_1^3 +$ $+\frac{1}{4}(2S_{11}+2S_{12}-S_{66})\cdot\sigma\cdot d_{\bar{6}(6)}(n_1-1)$
 $i = 6\,(\bar{6})$ $k = 3$ $m = 6$	π_{66}	$\delta\Delta_3 = -\frac{\pi_{11}+\pi_{12}\pm\pi_{66}}{4}\cdot\sigma\cdot d_3\cdot n_1^3 +$ $+S_{13}\cdot\sigma\cdot d_3(n_1-1)$
 $i = 6'$ $k = \bar{6}'$ $m = 6'$ 1) $\alpha=\alpha_1\neq 45°$ 2) $\alpha=\alpha_2\neq 45°$ $\alpha_1\neq\alpha_2$	π_{16} π_{61}	$\delta\Delta_{\bar{6}'} = -\frac{1}{2}\sigma\cdot d_{\bar{6}'}\cdot n_1^3[\pi_{11}(1+\tan^4\alpha)+2(\pi_{12}+\pi_{66})\tan^2\alpha+(\pi_{16}+2\pi_{61})\times$ $\times(\tan^3\alpha-\tan\alpha)]+\sigma\cdot d_{\bar{6}'}\cos^4\alpha[(2S_{11}-S_{66})\tan^2\alpha+S_{12}(1+\tan^4\alpha)-$ $-2S_{16}(\tan^3\alpha-\tan\alpha)](n_1-1).$
$i = 4$; $k = \bar{4}$; m $\perp$B 	π_{45}	$\delta\Delta_{\bar{4}} = -\sqrt{2}\,\frac{\sigma\cdot d_{\bar{4}}}{8(B_1+B_3)^{3/2}}[\pi_{11}+\pi_{13}+3\pi_{31}+\pi_{33}+2(\pi_{12}+\pi_{44})+\sqrt{2}(2\pi_{45}-\pi_{16})]+$ $+\frac{1}{8}\sigma\cdot d_4[S_{11}+S_{33}-S_{44}+2(S_{12}+2S_{13})-\sqrt{2}S_{16}](\frac{\sqrt{2}}{\sqrt{B_1+B_3}}-1)$

Table 2.6. Tetragonal system (classes 422, 4mm, $\bar{4}2m$ and 4/mmm).

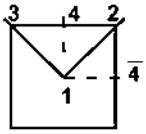
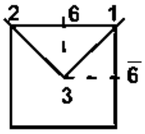
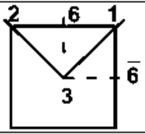
 $i = 4\,(\bar{4})$, $k = \bar{4}\,(4)$, $m = 4\,(\bar{4})$	π_{44}	$\delta\Delta_{\bar{4}(4)} = -\sqrt{2}\dfrac{\pi_{11}+\pi_{13}+\pi_{31}+\pi_{33}+2\pi_{44}}{4(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{\bar{4}(4)} + \dfrac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{13})\cdot\sigma\cdot d_{\bar{4}(4)}\left(\dfrac{\sqrt{2}}{\sqrt{B_1+B_3}}-1\right)$
 $i = 6\,(\bar{6})$, $k = \bar{6}\,(6)$, $m = 6\,(\bar{6})$	π_{66}	$\delta\Delta_{\bar{6}(6)} = -\dfrac{\pi_{11}+\pi_{12}+\pi_{66}}{4}\cdot\sigma\cdot d_{\bar{6}(6)}\cdot n_1^3 + \dfrac{1}{4}(2S_{11}+2S_{12}-S_{66})\cdot\sigma\cdot d_{6(6)}(n_1-1)$
 $i = 6\,(\bar{6})$, $k = 3$, $m = 6$	π_{66}	$\delta\Delta_3 = -\dfrac{\pi_{11}+\pi_{12}\pm\pi_{66}}{4}\cdot\sigma\cdot d_3\cdot n_1^3 + S_{13}\cdot\sigma\cdot d_3(n_1-1)$

Table 2.7. Hexagonal system (classes 6, $\bar{6}$ and 6/m).

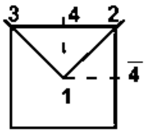
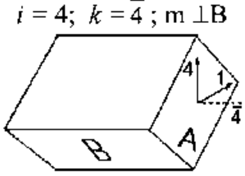
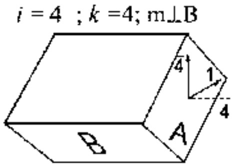
 $i = 4\,(\bar{4})$, $k = \bar{4}\,(4)$, $m = \bar{4}\,(4)$	π_{44}	$\delta\Delta_{4(4)} = -\sqrt{2}\dfrac{\pi_{11}+\pi_{13}+\pi_{31}+\pi_{33}+2\pi_{44}}{4(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{4(4)} + \dfrac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{13})\cdot\sigma\cdot d_{\bar{4}(4)}\left(\dfrac{\sqrt{2}}{\sqrt{B_1+B_3}}-1\right)$
$i = 4$; $k = \bar{4}$; m $\perp$B 	π_{45} π_{16}	$\delta\Delta_{\bar{4}} = -\sqrt{2}\dfrac{\sigma\cdot d_{\bar{4}}}{8(B_1+B_3)^{3/2}}[\pi_{11}+\pi_{33}+\pi_{13}+3\pi_{31}+2(\pi_{12}+\pi_{44})+\sqrt{2}(2\pi_{45}-\pi_{16})] + \dfrac{\sigma\cdot\delta_4}{8}[S_{11}+S_{33}-S_{44}+2(S_{12}+2S_{13})]\cdot\left(\dfrac{\sqrt{2}}{\sqrt{B_1+B_3}}-1\right)$
$i = \bar{4}$; $k = 4$; m$\perp$B 	π_{45} π_{16}	$\delta\Delta_4 = -\sqrt{2}\dfrac{\sigma\cdot d_4}{8(B_1+B_3)^{3/2}}[\pi_{11}+\pi_{33}+\pi_{13}+3\pi_{31}+2(\pi_{12}-\pi_{44})+\sqrt{2}(2\pi_{45}+\pi_{16})] + \dfrac{\sigma\cdot d_4}{8}[S_{11}+S_{33}-S_{44}+2(S_{12}+2S_{13})]\cdot\left(\dfrac{\sqrt{2}}{\sqrt{B_1+B_3}}-1\right)$

Table 2.8. Hexagonal system (classses 622, 62m, 6mm and 6/mmm)

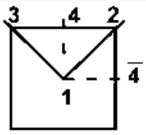
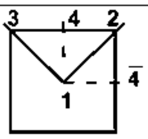
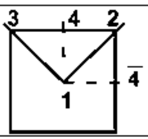
 $i = 4\,(\bar{4})$, $k = \bar{4}\,(4)$, $m = 4\,(\bar{4})$	π_{44}	$\delta\Delta_{\bar{4}(4)} = -\sqrt{2}\dfrac{\pi_{11}+\pi_{13}+\pi_{31}+\pi_{33}+2\pi_{44}}{4(B_1+B_3)^{3/2}}\cdot\sigma\cdot d_{\bar{4}(4)} + \dfrac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{13})\cdot\sigma\cdot d_{\bar{4}(4)}\left(\dfrac{\sqrt{2}}{\sqrt{B_1+B_3}}-1\right)$

Table 2.9. Cubic system

 $i = 4\,(\bar{4})$, $k = \bar{4}\,(4)$, $m = 4\,(\bar{4})$	π_{44}	$\delta\Delta_{4(4)} = -\dfrac{2\pi_{11}+\pi_{12}+\pi_{13}+2\pi_{44}}{8}\cdot\sigma\cdot d_{4(4)}\cdot n_1^3 + \dfrac{1}{4}(S_{11}+S_{33}-S_{44}+2S_{13})\cdot\sigma\cdot d_{\bar{4}(4)}(n_1-1)$
 $i = 4\,(\bar{4})$, $k = 1$, $m = 4$	π_{44}	$\delta\Delta_1 = -\dfrac{2\pi_{11}+\pi_{12}+\pi_{13}\pm 2\pi_{44}}{8}\cdot\sigma\cdot d_1\cdot n_1^3 + S_{12}\cdot\sigma\cdot d_1(n_1-1)$
Note: for cubic classes 432, $\bar{4}3m$ and m3m, $\pi_{12} = \pi_{13}$.		

Note for all tables: if in the formula near the sought for π_{im} there is the sign "±" for direct and symmetrical experiment conditions, that is, two formulae for $\delta\Delta_k$ are brought together in one, then subtracting these formulae we obtain essentially simpler expression to determinate the sought for π_{im}.

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