
Method of Light Beam Orbital Angular Momentum Evaluation by Means of Space-Angle Intensity Moments

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Abstract

Value of the orbital angular momentum (OAM) of an optical beam can be determined through the structure of matrix of its space-angle intensity moments. Considering the properties of these moments and their transformations, a conclusion has been made that any light beam with the OAM experiences a characteristic transverse shift during its interaction with a plane refracting boundary or a diffraction grating. On this base, a method for the immediate measurement of the beam OAM is proposed. The simple experimental arrangement for such a measurement includes a self-collimating diffraction grating, a position-sensitive quadrant photodetector and a device for the beam rotation around its longitudinal axis.

Key words: optical vortex, angular momentum, intensity moments, dispersion elements, beam shift, diffraction grating.

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1. Introduction

The existence of an "orbital" (independent on the state of polarisation) mechanical angular momentum (OAM) is a general feature of light beams with optical vortices [1,2]. Therefore, the measurement of the OAM composes an important task. Measurement methods used so far [3,4] employ the mechanical action of the beam with OAM, exerted on miniature particles, and their accuracy is rather low because of very small absolute values of immediately measured mechanical effects. Besides, such measurements cannot separate contributions of "spin" (stipulated by the circular polarisation) and orbital angular momenta, as well as rotational action caused by the "propeller effect". At the same time, the detailed registration of the phase and amplitude distribution over the beam cross-

section, which would allow determining the OAM in a "direct" way, is an extremely difficult problem.

In this work, a new approach is proposed, that is grounded on the straight determination of the light beam spatial structure, responsible for its OAM. Recently it has been shown [2] that the presence of the OAM is immediately reflected in the structure of matrix of the beam space-angle intensity moments [5]. In its turn, this structure affects the beam behaviour at its interaction with a plane dispersive element (e.g., refracting boundaries or diffraction gratings) [6,7]. This influence leads to certain transformations in the spatial shape of the passed beam, but the most interesting fact is that in some cases remarkable relations exist between the moments and the values of well-defined characteristics of the transformed beam,

such as the position and direction of the beam "centre of gravity". Due to these relations a straight and simple measurement of the moment matrix elements, including mixed space-angle moments, becomes available [7]. A combination of these ideas in application to beams with OAM is the main subject of the present work.

2. Intensity moments and their transformation

The matrix of second moments of the space-angle intensity distribution (intensity moments) was introduced for suitable characterisation of the energy distribution over paraxial light beams [5]. Generally, the 4×4 symmetric matrix of intensity moments (usually represented through four 2×2 blocks) can be defined as

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \frac{1}{\Phi} \int \begin{pmatrix} \mathbf{r} \\ \mathbf{p} \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{r}} & \tilde{\mathbf{p}} \end{pmatrix} I(\mathbf{r}, \mathbf{p}, z) (dr) (dp) \quad (1)$$

Here Φ is the total energy flux (power) of the beam propagating along axis z , $\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ is the column vector of transverse spatial coordinates, $\mathbf{p} = \begin{pmatrix} p_x \\ p_y \end{pmatrix} = n \frac{d\mathbf{r}}{dz}$ (n is the refraction index) denotes the vector of spatial frequencies (angular ray co-ordinates) [8], symbol \sim indicates a matrix transposition, $(dr) = dx dy$, and

$$I(\mathbf{r}, \mathbf{p}, z) = \frac{1}{\lambda^2} \int u \left(\mathbf{r} + \frac{\mathbf{r}'}{2}, z \right) u^* \left(\mathbf{r} - \frac{\mathbf{r}'}{2}, z \right) \times \exp[-ik(\mathbf{p} \cdot \mathbf{r}')] (dr')$$

is the Wigner distribution function [9] of a light beam with wavelength $\lambda = 2\pi/k$ and the complex amplitude distribution $u(\mathbf{r}, z)$ normalized so that $\int |u(\mathbf{r}, z)|^2 (dr) = \Phi$. Initially, the moments, constituting matrix (1), were considered without any connection to vortex beams, but it was subsequently established

[2,10] that the anti-symmetrical part of 2×2 matrices

$$M_{12}(z) = \tilde{M}_{21}(z) = \begin{pmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{pmatrix} \quad (2)$$

(off-diagonal blocks of the full moment matrix (1)), can characterise the linear density of the beam OAM Λ (the OAM of the radiation contained within a unit length of the beam "body"):

$$\Lambda = \frac{\Phi}{c^2} (m_{xy} - m_{yx}) \quad (3)$$

where c is the light velocity.

This conclusion allows to employ the powerful technique of the intensity moment matrix and its transformations [5] in studying the beams with OAM. Such possibilities arise in the connection to the beam transformations during its passage through plane dispersion elements (PDE) – diffraction gratings or/and refracting boundaries [6,7,11] (see Fig. 1).

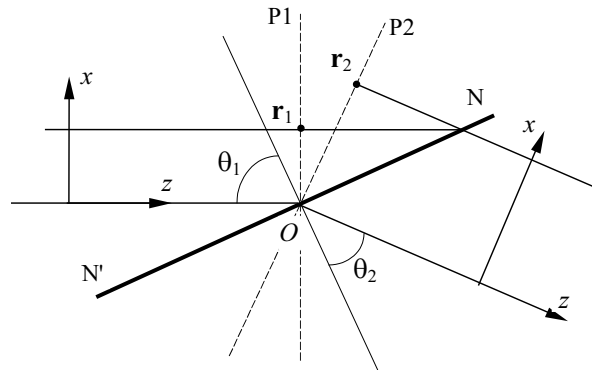


Fig. 1. General schematic of a PDE

Here $N'N$ is the trace of a PDE plane (boundary or grating plane), before and after which homogeneous media with refraction indices n_1 and n_2 are situated; θ_1 and θ_2 are angles of incidence and diffraction, respectively (in the case of reflection $\theta_2 > \pi/2$). The beam propagates along axis z . The fracture of axis z in the point of incidence O reflects the beam deviation due to the refraction or diffraction and

allows to use the model of paraxial beam propagation [8] along its whole trajectory (both before and after PDE); axis x is orthogonal to axis z and y is orthogonal to the figure plane. The input (before PDE) and output (after PDE) spatial parameters of the beam relate to the input and output reference planes [12] P_1 and P_2 , which are normal to the input (output) segments of axis z and intersect the PDE plane in point O .

In geometrical optics, the system of Fig. 1 executes a projective transformation, when the output beam profile merely reproduces the input one with possible scale change along axis x [13]. It means that distributions of the beam complex amplitude in the input reference plane $u_1(\mathbf{r})$ and in the output one $u_2(\mathbf{r})$ are related by the simple geometric transformation $u_2(\mathbf{r}) \propto u_1(\tilde{\mathbf{D}}\mathbf{r})$, where \mathbf{D} is a certain transformation matrix [6,13]. Therefore, an arbitrary characteristic point of the input distribution (\mathbf{r}_1 in plane P_1) has its counterpart $\mathbf{r}_2 = \tilde{\mathbf{D}}^{-1}\mathbf{r}_1$ in plane P_2 . Particularly, this is true for the beam "centre of gravity" (CG)

$$\mathbf{r}_0 = \frac{1}{\Phi} \int \mathbf{r} I(\mathbf{r}, \mathbf{p}) (dr)(dp).$$

But the allowance for the diffraction in PDE [6,7,11] reveals certain distortions of the beam shape during its passage through the system of Fig. 1. In consequence, the CG of the output beam experiences a shift with respect to the position predicted by the geometrical consideration. And, what is the most important, this shift appears to be dependent on the elements of matrix (2) [7]:

$$\begin{aligned} \Delta \mathbf{r}_0 = & -\frac{1}{n_2} \mathbf{D} \mathbf{M}_{21} \mathbf{D}^{-1} \begin{pmatrix} \tan \theta_2 \\ 0 \end{pmatrix} + \\ & + \frac{1}{n_1} \tilde{\mathbf{D}}^{-1} \mathbf{M}_{21} \begin{pmatrix} \tan \theta_1 \\ 0 \end{pmatrix} + \\ & + 2 \tilde{\mathbf{D}}^{-1} \mathbf{M}_{12} \mathbf{f}' - \frac{1}{k} \tilde{\mathbf{D}}^{-1} \mathbf{f}'' \end{aligned} \quad (4)$$

This deviation of the CG position originates from the fact of finite width of the angular

spectrum of a spatially limited beam [6,7]. In equation (4), $\mathbf{f} = \mathbf{f}' + i\mathbf{f}'' = \nabla_{\mathbf{p}} [\ln \tau(\mathbf{p})]$, where $\tau(\mathbf{p})$ is the "amplitude efficiency" of a plane wave transformation accounting amplitude changes upon the wave passing through the PDE [6]. Therefore, the third and fourth terms describe the role of variable PDE transmission for different angular components of the beam (note that in the earlier analysis [7] the restriction $\mathbf{f}' = 0$ was implied, which now is cancelled). The last term of (4) corresponds to the effect whose nature is similar to known Goos – Hänchen shift [14] and its magnitude does not depend on the beam shape. But the other ones provide connections to the OAM of the considered beam.

3. Principle of the OAM measurement

In the most common case, when the PDE is formed by a smooth refracting boundary or by a grating with grooves normal to axis z , the transformation matrix \mathbf{D} equals to [7,11]

$$\mathbf{D} = \begin{pmatrix} \cos \theta_1 & 0 \\ \cos \theta_2 & 0 \\ 0 & 1 \end{pmatrix}. \quad (5)$$

Furthermore, it follows from the symmetry considerations [7] that in this case $\tau(\mathbf{p})$ is an even function with respect to p_y and, consequently, $\mathbf{f} = \begin{pmatrix} f_x \\ 0 \end{pmatrix}$. Combining (2), (4) and (5), one can easily find the Cartesian components of the CG shift (4):

$$\Delta x_0 = m_{xx} v_x + \frac{\cos \theta_2}{\cos \theta_1} \left(2m_{xx} f'_x - \frac{1}{k} f''_x \right) \dots, \quad (6)$$

$$\Delta y_0 = m_{xy} v_y + 2m_{yx} f'_x$$

where

$$\begin{aligned} v_x = & \frac{1}{n_1} \frac{\sin \theta_1 \cos \theta_2}{\cos^2 \theta_1} - \frac{1}{n_2} \tan \theta_2 \dots \\ v_y = & \frac{1}{n_1} \tan \theta_1 - \frac{1}{n_2} \frac{\sin \theta_2}{\cos \theta_1} \end{aligned} \quad (7)$$

It is seen from (6) that the existence of a non-zero OAM (3) can only be manifested in the

transverse (normal to the plane of incidence) component of the CG shift Δy_0 . This feature is favourable, permitting to distinguish the OAM contribution from effects of different nature, which produce longitudinal CG displacements. Nevertheless, the "substantial" (f_x) and "geometrical" (v_x, v_y) factors act, in general, jointly, though differently, and can mask each other in the course of measurements. Therefore, situations, where the results will be unambiguous, should be looked for.

In the case of specular reflection ($\theta_2 = \pi - \theta_1, n_2 = n_1$), the geometrical factors vanish ($v_x = v_y = 0$), and the CG shift, predicted by formulae (6), (7), is fully conditioned by f_x , that is, by the optical properties of the contacting media. In this case, possible longitudinal (within the plane of incidence) beam displacement $\Delta x_0 = k^{-1} f_x'' - 2m_{xx} f_x'$ consists of two parts. The first part is associated with variations of the reflection coefficient phase and is identical to the Goos – Hänchen shift, the second one is conditioned by the angle dispersion of the absolute value of reflection. Although the latter carries, through m_{xx} , certain dependence on the beam structure, it has no connection with the OAM.

A transverse shift of CG $\Delta y_0 = 2m_{yx} f_x'$ can also appear in this case. A special example of this effect, dealing with circular Laguerre-Gaussian (LG) beams, has been predicted recently [15,16]; the present consideration shows its relation to the general theory of the beam transformations in the PDE and reveals a especial role of the beam OAM. Due to the dependence on m_{yx} , this shift can be used, in principle, for measurements of the OAM linear density (3). At the same time, strong influence of the media's properties, expressed by multiplier f_x' , puts its own restriction and makes corresponding measurement possibilities

rather poor.

Meanwhile, the special peculiarities of a PDE geometry provide, in general, much larger variety of potential measurement advantages. Even upon the refraction at a smooth surface ($n_1 \sin \theta_1 = n_2 \sin \theta_2$) the geometrical factors differ from zero:

$$v_x = \frac{1}{n_1} \frac{\tan \theta_1}{\cos \theta_1 \cos \theta_2} \left[1 - \left(\frac{n_1}{n_2} \right)^2 \right],$$

$$v_y = \frac{1}{n_1} \tan \theta_1 \left[1 - \left(\frac{n_1}{n_2} \right)^2 \right],$$

and the CG shift, stipulated by the first and second terms of (4), exists and can be measured. However, it becomes the most noticeable in case of self-collimating reflection from an oblique grating ($\theta_2 = \theta_1 + \pi$). Adopting for the simplicity $n_2 = n_1 = 1$ in (7), we will get in this situation

$$v_y = -v_x = 2 \tan \theta_1$$

and under quite possible condition of grazing incidence ($\tan \theta_1 \gtrsim 10$), a high enough CG shift can be observed [7]. The use of the reflecting echelle gratings [17] is especially suitable because, in the usual mode of operation, the input beam approaches normally the facets of grooves, which corresponds to the condition $f_x = 0$ [7]. Therefore, the results of the shift measurements appear to be free from the "substantial" influence and allow to determine the moment matrix elements unambiguously:

$$\Delta x_0 = -2m_{xx} \tan \theta_1,$$

$$\Delta y_0 = 2m_{yx} \tan \theta_1. \quad (8)$$

Also, the case of a self-collimating grating is suitable for the visual demonstration of the nature of the considered beam shift, at least in its "geometrical" part (see Fig. 2). In this situation, the input and output reference planes P_1, P_2 coincide and form the common reference plane. Within this plane, the trace of the input beam, propagating to the right (along the axis z which is not shown in Fig. 2), is denoted by the

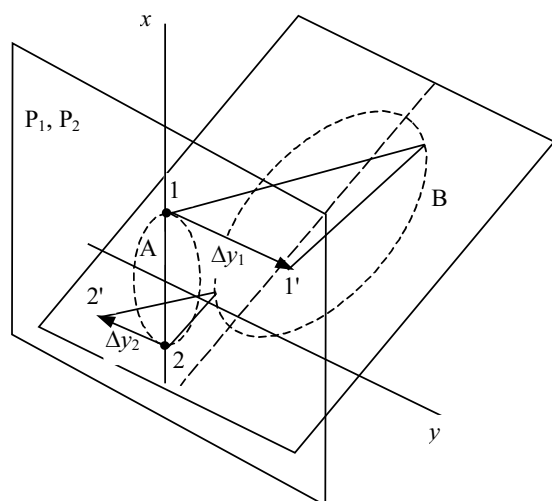


Fig. 2. The nature of the beam transverse shift at the self-collimating reflection

dashed circle A. The grating plane is inclined, and the beam trace B in this plane has approximately an elliptical shape. If the beam has the helical wave front (which is a typical feature of beams carrying OAM), the energy flux possesses a certain vortex transverse component. For example, the energy, localised near some point 1 of the input beam cross section, propagates with some deviation to the side of positive y , while the energy from point 2 will deviate in the opposite transverse direction. Further, between the reference plane and the grating, the energy propagates along the straight rays, which are reflected in the points of intersection with the grating. The reflection is self-collimating in xz plane and specular in zy plane. Hence, the energy which was initially localised at point 1, in the output beam will be localised near point 1', with some transverse shift Δy_1 , and the energy from point 2 will be transferred to point 2' with corresponding shift $-\Delta y_2$. Since the distance between the reference plane and the grating is higher for point 1 than for point 2, the inequality $\Delta y_1 > \Delta y_2$ takes place. Obviously, the same consideration can be made for all pairs of diametrically opposite points of the input beam cross section, and for every small portion of the beam energy, which is shifted to the positive y (from the upper half of the

section) there the equivalent portion, will be found which is shifted to negative y , but to a minor degree. As a result, the whole profile of the reflected beam is effectively “displaced” in the y direction.

4. Scheme of the OAM measurement

Turning back to the measurement problems, we should note that, following to (6) – (8), the CG shift measurements can be used for the determination of only one moment m_{xy} , which, generally, gives no access to the beam OAM via (2) and (3). The value of m_{yx} should also be measured. For this purpose, the properties of the moment matrix transformations [5] will be expedient. For example, if the beam complex amplitude distribution is transformed in accord of the law

$$u(\mathbf{r}) \rightarrow u(\mathbf{T}\mathbf{r}) \quad (9)$$

with 2×2 transformation matrix \mathbf{T} , the moment matrix (2) experiences the modification $M_{12} \rightarrow M'_{12} = \mathbf{T}M_{12}\mathbf{T}^{-1}$. In particular, we can require that

$$M'_{12} = \begin{pmatrix} m'_{xx} & m'_{xy} \\ m'_{yx} & m'_{yy} \end{pmatrix} = \begin{pmatrix} m_{yy} & m_{yx} \\ m_{xy} & m_{xx} \end{pmatrix};$$

such a transformation corresponds to the reciprocal substitution of co-ordinates x and y and is described by the matrix

$$\mathbf{T} = \pm \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (10)$$

Obviously, in the same conditions, the shift Δy_0 for the transformed beam will be proportional to $m'_{xy} = m_{yx}$, which will allow to determine the lacking matrix element m_{yx} of the initial beam. By the way, it is worthwhile to remark that each of the moments m_{xy} , m_{yx} depends on the choice of co-ordinate axes, but their difference is, of course, invariant [5].

Therefore, the measurement of the beam OAM would be completed if two experimental

problems are solved: determining the transverse shift of the beam CG and transforming the beam according to the transformation matrix (10). Let us begin with the second one and consider the physical implementation of the transformation required. One can easily see that it can be obtained combining the reflection and rotation of the initial beam profile, and several ways to perform this are available. In our work, it was realised by means of an optical system analogous to the mode converter, recently proposed for the LG and Hermite-Gaussian mode conversion [18] (Fig. 3). In this device, flat mirrors M1 and M3 are semitransparent, which allows to split an input beam into two arms and to have different beam structures at the output, depending on which channel, 1 or 2 (or both), is open. All the mirrors M1, M2 and M3 are parallel to axis y' , lying in the xy plane, and are aggregated in a hard construction that can rotate, as a whole, around axis z .

At firsting, consider the beam passed the channel 2; let its complex amplitude distribution at the input of the measuring unit 5 be $u(\mathbf{r})$. Then, if the same input beam passes the channel 1, its profile near the measuring unit will accept the form (9) with the transformation matrix [19]

$$T = \begin{pmatrix} -\cos 2\Theta & -\sin 2\Theta \\ -\sin 2\Theta & \cos 2\Theta \end{pmatrix},$$

where Θ is the angle between axes y and y' , i.e. the angle of rotation of the mirror system relatively to the co-ordinate frame that was initially chosen within the beam cross-section. This transformation matrix has the required form (10) at $\Theta = \pm\pi/4$.

The most appropriate scheme of the measuring unit 5 contains a self-collimating diffraction grating as a main element (Fig. 4). In this situation, the grating grooves are parallel to y axis (normal to the figure plane), which ensures the conditions for the applicability of equations (8). Really, it is the grooves that determine the choice of initial co-ordinate axes, and their orientation dictates the orientation of all other parts of the measuring device, especially of the mirror system in Fig. 3. The scatterer 9 provides the diffuse "broadening" of the beam, which enables to measure the CG position directly by means of a quadrant photodetector [20]; of course, the use of advanced beam analysers [21] is also possible.

In general case, two beams fall upon the photodetector (par excellence, of the quadrant type with quadrants oriented in accordance to axes x, y) sensitive area: one reflected by the grating (whose transverse shift, properly, should be measured) and one reflected by the mirror 8 (the reference beam). The last one serves for the

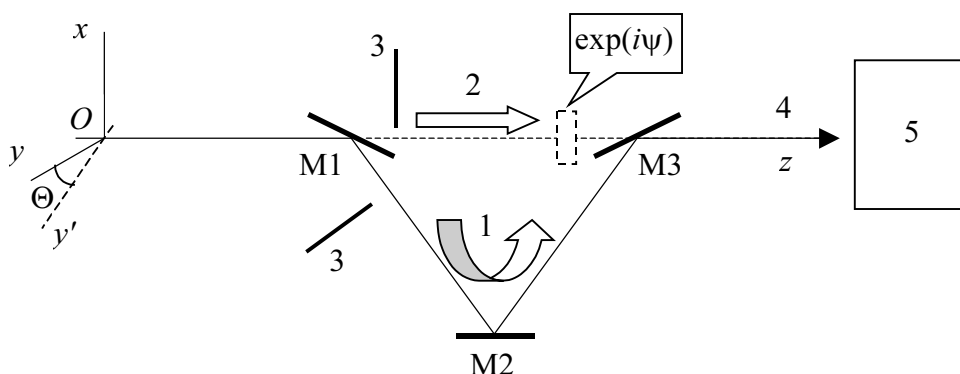


Fig. 3. Schematic of the mode converter: 1, 2 – arms (channels of the beam passage), 3 – movable shutter, 4 – output beam, 5 – measuring unit.

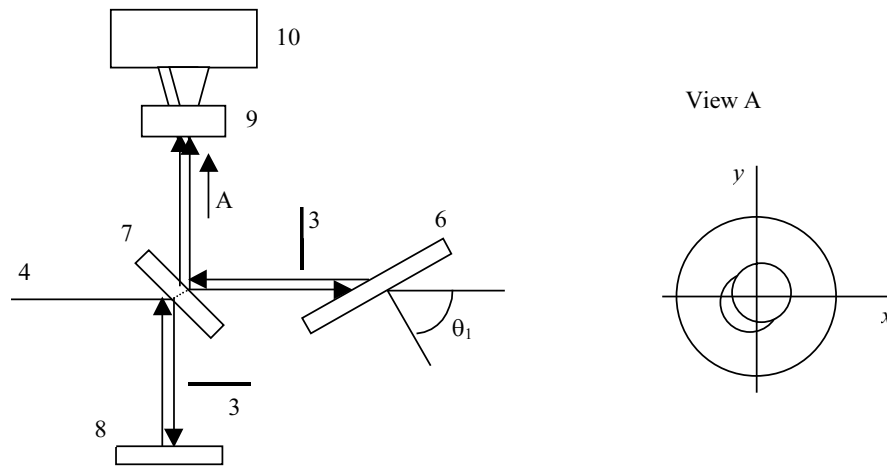


Fig. 4. Structure of the measuring unit (numeration in Fig. 3 is continued): 3 – shutters, 4 – analysed beam, 6 – grating, 7 – light splitter, 8 – flat mirror, 9 – diffuse scatterer, 10 – position-sensitive photodetector, whose light-sensitive area is depicted in separate view A (the actual beam deformation is not shown and only its shift is indicated schematically).

system alignment and calibration: if the incoming beam 4 carries no OAM, both spots on the photodetector window must coincide (more exactly, they must experience the same transverse shift of CG).

In the course of the measurement procedure, at first, the channel 1 is locked by the shutter 3 (Fig. 3). Consequently, only the beam with the initial (non-transformed) structure reaches the measuring unit. Then the mirror 8 and the grating 6 are screened alternatively by their shutters, and in both cases the photodetector signals are registered; their difference is just proportional to the transverse shift Δy_0 of the beam, reflected by the grating. Hence, through the second equation (8), m_{xy} is determined. Afterwards, the channel 1 is being opened and the channel 2 becomes locked, so that only the transformed beam can reach the unit 5; then, the same manipulations again give the value of CG shift that now is proportional to m_{yx} . Finally, the total OAM of the beam is found by the formula (3).

5. The case of a circular LG mode

These techniques may be farther developed and simplified in application to circular LG beams

[1,22]. In this case, matrix (2) is antisymmetric (this follows from the definition of the intensity moments (1) and, e.g., from the results of Appendix A in Ref. 5). It means, foremost, that $m_{xx} = 0$ and, due to (6), $\Delta x_0 \propto f_x''$; therefore, in the case of self-collimating reflecting grating, when $\mathbf{f} = 0$ (see Section 3), such a beam experiences only a transverse (y -directed) shift. Secondly, $m_{xy} = -m_{yx}$, and the simple application of formulae (3), (6) and (8) directly gives

$$\Delta y_0 = \Lambda \frac{c^2}{2\Phi} v_y = \Lambda \frac{c^2}{\Phi} \tan \theta_1. \quad (11)$$

Thus, if an LG beam is diffracted by a grating, its CG undergoes the transverse displacement whose magnitude is proportional to the linear density of the OAM Λ . Remembering that for an LG beam with azimuthal index l

$$\Lambda = l \frac{\Phi}{kc^2} = l \frac{\lambda \Phi}{2\pi c^2}$$

(see, e.g., [1,2]), one can represent the result (11) in the form

$$\Delta y_0 = l \frac{\lambda}{4\pi} v_y = \frac{l}{k} \tan \theta_1. \quad (12)$$

Due to relations (11), (12) the investigation of the OAM of an LG beam is a simpler task, in

comparison with the general case, because the manipulations with the transformed beam are not necessary. In this circumstance, the whole measurement can be completed using only one of the two possible channels for the beam passage in Fig. 3. But properties of the mode converter structure [18] provide some additional experimental possibilities. Two following demonstrations seem to be rather attractive:

1. If the input beam, entering the optical system in Fig. 3, has the LG structure, then, depending on which channel, 1 or 2, it passes, the spirality [1,2] (i.e., the mode index l) of the output beam 4 will change its sign. Therefore, toggling the channels of the beam passage alternatively, the photodetector will generate an AC signal whose magnitude is proportional to the beam OAM.
2. If the input beam has the Hermite-Gaussian structure and is properly orientated with respect to axes x , y , and both channels are open, then the beam 4 will carry an OAM with magnitude depending on the phase shift ψ between the channels [18]. Consequently, by changing this phase with a phase corrector, placed in one of the channels (e.g., 2 in Fig.2) and without other changes in the system, one can observe the modulation of the photodetector signal synchronously with the modulation of the output beam OAM.

6. Conclusion

The direct relation between the OAM of an optical beam and its moment matrix, demonstrated in this work, allowed to include the OAM concept into the well developed scheme [23] of a laser beam characterisation by the second space-angle intensity moments with all the accompanying benefits: commercially available equipment, software, etc. In this context, a conclusion about the CG shift, appearing when a light beam with OAM propagates through a PDE, is only the first step and we hope that further applications of this scheme will bring new helpful prospects.

The main advantage of the OAM measurement method, proposed in this paper, is that the whole procedure is completely optical one, with no unreliable mechanical transformation, and it allows the use of advanced means of the beam measurement and characterisation. In comparison with the top achievements [24], the experimental arrangement for the OAM measurement, described in the present work, may seem not so much elaborated, but it has obvious practical profits. The self-collimating scheme supplies simple and exact means for the comparative measurement of the beam distortion at the diffraction grating, the use of a quadrant photodetector with a diffuse beam scatterer provides easy way for the CG position measurement without complicated numerical processing, and the mode converter executes necessary beam transformation in the most natural manner. This set-up contains no movable parts (except mode of operation switches) and does not require microscopic observations; it allows simple adjustment and re-arrangement and can be supplemented by modern beam analysers with corresponding software.

Finally, it should be emphasised that throughout the paper a scalar wave approximation was used; for electromagnetic waves it is proved if the beam plane polarisation properly agrees with a PDE geometry and does not change during the beam transformations [7]. If this condition does not hold and some elliptical polarisation appears, an additional, so called "spin-dependent" [16] beam shift may take place (see, e.g., [25-28] and references therein), which can affect the OAM measurements to some extent. Nevertheless, this influence is not crucial and can be avoided. Firstly, the spin-dependent shift normally does not exceed some parts of the light wavelength while the OAM contribution can be much more [16] (for example, it is quite possible to have $\tan\theta_1 \sim 10$ in Eq. (12)); besides, the OAM-stipulated effect can be multiplied for beams

with high OAM (in Eq. (12) this corresponds to $l > 1$) while the spin-dependent one cannot. Secondly, the latter can be removed at all if the incident beam polarisation is strictly parallel or orthogonal to the incidence plane [16,17] (in the case of diffraction grating this means that the beam polarisation parallel or orthogonal to grooves should be maintained).

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