Non-reciprocal acoustooptic effects in gyrotropic cubic crystals with electroinduced anisotropy

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Abstract

In the present report non-reciprocal effects at acoustooptical interaction in gyrotropic cubical crystal with electroinduced anisotropy have been considered. It has been shown that the existence of optical gyrotropy causes the doubling of maxima of amplitude non-reciprocity. Application of the electric field leads to displacement and increase of maxima of amplitude non-reciprocity. The possibility of switching the direction of laser generation of ring laser by means of the reversing of applied electric field has been established. Obtained results may be used for creation and optimization of parameters of non-reciprocal elements with electric and polarization controlling.

Keywords: non-reciprocity, gyrotropy, induced anisotropy, acoustooptical effect

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Introduction

One of the actual problems of modern laser physics is obtaining single-frequency lasing. In this case the non-reciprocal effects, caused by difference of amplitudes and phases of waves passing the optical element in the ring resonator in contrary directions, are widely used. However, use of the traditional Faraday elements, when the presence of non-reciprocity is caused by the influence of external magnetic field, is not expedient in the case of powerful solid state infrared lasers because of the absence of materials with high magnetooptic activity in the given spectrum range.

In the last years the opportunity of obtaining single-frequency lasing with the help of diffraction of contrary light waves on ultrasonic wave is shown theoretically and experimentally [1-6]. In general the peculiarities of acoustooptical (AO) interaction were investigated in cubic non-gyrotropic crystals. Nevertheless, there are some gyrotropic crystals (for example $Bi_{12}GeO_{20}$, $Bi_{12}SiO_{20}$ and others), which possess high AO effectivity. In the papers [7,8] it was shown that existence of gyrotropy caused doubling and considerable displacement of maxima of non-reciprocity. This is important for the obtaining of the conditions of one-directional lasing. In the present report non-reciprocal effects at AO interaction in cubic gyrotropic crystals with electroindused anisotropy are investigated.

Theoretical consideration

Let us consider the interaction geometry, with sound wave

$$\mathbf{u} = \mathbf{u}_0 \exp i(Kx - \Omega t) \qquad (1)$$

where K is wave vector, $\Omega = K\upsilon$ - frequency (υ is the phase velocity) and $\mathbf{u}_0$ - ultrasonic wave amplitude that propagated along x axis (we suppose, that x axis is collinear to [100] axis of the 23 class crystal). Plane light wave propagate at small angle Θ to z axis (axis [001]). It will be assumed, that the interaction range of light and sound waves is limited by planes $z=0$, $z=l$, where l is the crystal thickness. We suppose also, that the external electrical field $\mathbf{E}^0$ is applied along [001] axis of a cubic gyrotropic crystal.

For the calculation of diffracting fields we shall use Maxwell equations and constitutive law [9] for gyrotropic crystal in an external electrical field, from which we may obtain the wave equation for tension of light field in the area held by ultrasonics:

$$\nabla^2\mathbf{E} - \frac{\varepsilon + \Delta\varepsilon^{el}}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} - \frac{2\alpha}{c^2}\frac{\partial^2}{\partial t^2} rot\mathbf{E} = \frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{P}, \qquad (2)$$

where ε is the dielectric constant of undisturbed media; α is scalar, related to the gyrotropic parameter γ and to the specific rotation ρ in the direction of polarization of light in crystal ($\rho = \omega\gamma/c = \omega^2\alpha/c^2$); ω is the frequency of the incident optical wave; $\Delta\varepsilon_{ij}^{el} = -\varepsilon^2 r_{ijk} E_k^o$ is the change of dielectric tensor caused by the presence of an external electrical field; r_{ijk} are electrooptic constants; E_k^o are components of the quasi-stationary electrical field applied to media; $\mathbf{P} = (\Delta\varepsilon + \Delta\varepsilon^*)\mathbf{E}/8\pi$ is induced electric polarization at the sum and the difference frequencies arising from the interaction of electromagnetic and acoustic waves on photoelastic nonlinearity; $\Delta\varepsilon_{ij} = -\varepsilon^2 p_{ijmn} u_{mn}$ is the tensor of perturbation of dielectric permeability of the crystal by the acoustic wave; p_{ijmn} is the tensor of photoelasticity; $u_{mn} = \frac{1}{2}(\partial u_m/\partial x_n + \partial u_n/\partial x_m)$ is the strain tensor.

The solution of a wave equation (2) will be searched in the form of two coupled waves with slowly varyed amplitudes [7,10]

$$\mathbf{E} = \mathbf{E}_0(z)exp\,i(\mathbf{k}_0\mathbf{r} - \omega_0 t) + \mathbf{E}_1(z)exp\,i(\mathbf{k}_1\mathbf{r} - \omega_1 t), \qquad (3)$$

where $\mathbf{E}_0(z) = \mathbf{e}_0 A_0(z) + \mathbf{e}_2 B_0(z)$; $\mathbf{E}_1(z) = \mathbf{e}_1 A_1(z) + \mathbf{e}_2 B_1(z)$; $\mathbf{k}_{0,1}$ are the wave vectors of the incidence (0) and diffracted (1) light waves; $\mathbf{e}_{0,1}$ are the corresponding unit vectors lying in the scattering plane perpendicular to the wave vectors of refracted and diffracted waves; $\mathbf{e}_2$ is a unit vector orthogonal to the acoustooptical interaction plane. So, the vectors $\mathbf{e}_2$, $\mathbf{e}_3$, $\mathbf{e}_{0,1}$ construct a right-handed triple. Let's note, that the influence of ultrasound, the external electric field and gyrotropy caused the changes of field vectors $\mathbf{E}_0(z)$, $\mathbf{E}_1(z)$. Substituting of the expression (3) into (2), one can obtain the system of equations, which can be presented in the matrix form:

$$\frac{dX}{dz} = MX, \qquad (4)$$

where

$$X = \begin{bmatrix} A_0 \\ B_0 \\ A_1 \exp i\Delta kz \\ B_1 \exp i\Delta kz \end{bmatrix};$$

$$M = \begin{bmatrix} i\kappa_{10}^{el} & \rho + i\kappa_{12}^{el} & i\kappa_{10} & i\kappa_{12} \\ -\rho + i\kappa_{20}^{el} & i\kappa_{22}^{el} & i\kappa_{21} & i\kappa_{22} \\ i\kappa_{10} & i\kappa_{12} & i\Delta k + i\kappa_{11}^{el} & \rho + i\kappa_{12}^{el} \\ i\kappa_{20} & i\kappa_{22} & -\rho + i\kappa_{21}^{el} & i\Delta k + i\kappa_{22}^{el} \end{bmatrix};$$

$\Delta k = (\mathbf{k}_1 - \mathbf{K} - \mathbf{k_0})\mathbf{e}_3$ is wave mismatch;

$$\kappa_{ij} = \frac{\pi}{2\lambda_0 n}\mathbf{e}_i \Delta\varepsilon \mathbf{e}_j, \quad \kappa_{ij}^{el} = \frac{\pi}{2\lambda_0 n}\mathbf{e}_i \Delta\varepsilon^{el}\mathbf{e}_j,$$

$i, j =$ 0, 1, 2; λ_0 is the wavelength of light in a vacuum; $n = \sqrt{\varepsilon}$. At the diffraction of light wave propagated along z axis, on transversal elastic wave polarized along y axis, one can write $\kappa_{12} = \kappa_{02} = \kappa_{21} = \kappa_{20} = \kappa = \overline{\beta} p_{44}$;

$$\overline{\beta} = -\frac{\pi n^3}{2\lambda_0}\sqrt{\frac{2I_a}{\sigma\upsilon^3}},$$

$\kappa_{12}^{el} = \kappa_{02}^{el} = \kappa_{21}^{el} = \kappa_{20}^{el} = \kappa^{el} = -\frac{\pi n^3}{2\lambda_0} r_{41} E_3$;

where I_a is the intensity of sound; σ is the density of media.

We shall consider non-reciprocal effects induced by acoustooptical interaction. Using the matrix method [11] and boundary conditions $A_0(0) = A_{\parallel}$, $B_0(0) = A_{\perp}$, $A_1(0) = B_1(0) = 0$ ($A_{\parallel}$, $A_{\perp}$ are imaginary amplitudes) none can obtain the solution of an equation (4)

$$X = \exp MzX_0, \qquad (5)$$

where $X_0 = (A_{\parallel}, A_{\perp}, 0{,}0)_t$ is the column vector of the boundary conditions; the index t means transposition.

At considered geometry of interaction (Fig. 1), the expression (5) can be presented in an analytical form. So, if the incident light has amplitude $\widetilde{E}_0$ and polarization vector $\widetilde{\mathbf{e}} = ((1+\tau)e^{-i\chi}\mathbf{e}_+ + (1-\tau)e^{i\chi}\mathbf{e}_-)/\sqrt{2(1+\tau^2)}$, where τ is ellipticity, χ is angle between the non-principal axis of an ellipse and x axis; $\mathbf{e}_\pm = \frac{1}{\sqrt{2}}(\mathbf{e}_0 \pm i\mathbf{e}_2)$, then the imaginary amplitude of the diffracted wave at the exit of the acoustooptic cell is given by the expression

$$\mathbf{E}_1(l) = \frac{\widetilde{E}_0}{\sqrt{2}}\{(S_1 - S_2)\mathbf{e}_0 + (S_1 + S_2)\mathbf{e}_2\} \times \exp\left(-i\frac{\Delta kl}{2}\right), \qquad (6)$$

$$S_{1,2} = \frac{\kappa}{2g}\left\{iq\kappa^{el\,2} + (1/2)ipg \pm v\kappa^{el}\right\} f_{2,1} \pm \frac{\rho}{2} q(2\kappa^{el} \mp \Delta k) f_{1,2},$$

$$p = \frac{1}{\beta_1}\sin\beta_1 l + \frac{1}{\beta_2}\sin\beta_2 l,$$

$$q = \frac{1}{\beta_1}\sin\beta_1 l - \frac{1}{\beta_2}\sin\beta_2 l,$$

$$v = \cos\beta_1 l - \cos\beta_2 l,$$

$$g = \sqrt{4\kappa^{el\,2}\kappa^2 + \rho^2\Delta k^2 + \kappa^{el\,2}\Delta k^2},$$

$$f_1 = \frac{1}{\sqrt{1+\tau^2}}\left\{(1+\tau)e^{-i\chi} + (1-\tau)e^{i\chi}\right\},$$

$$f_2 = \frac{i}{\sqrt{1+\tau^2}}\left\{(1+\tau)e^{-i\chi} - (1-\tau)e^{i\chi}\right\},$$

$$\beta_1 = \sqrt{\Delta k^2/4 + \kappa^2 + \rho^2 + \kappa^{el\,2} + g},$$

$$\beta_2 = \sqrt{\Delta k^2/4 + \kappa^2 + \rho^2 + \kappa^{el\,2} - g}.$$

The intensity I_d of the diffracted wave is given by the expression:

$$I_d = I^0\left[\frac{\kappa}{2g\sqrt{1+\tau^2}}\right]^2\Big[(1+\tau^2)(\rho^2q^2\Delta k^2 + g^2p^2 + 4\kappa^{el\,4}q^2 + 4\rho^2q^2\kappa^{el\,2} + 4\kappa^{el\,2}v^2 + 4\kappa^{el\,2}qgp) + (1-\tau^2)4\kappa^{el\,2}\rho q\Delta k(v\sin 2\chi - \rho q\cos 2\chi) + 4\rho\Delta kq(pg + 2q\kappa^{el\,2})\tau\Big], \qquad (7)$$

where I^0 is the intensity of incident electromagnetic radiation.

As one can see from (6), the diffracted wave possesses elliptic polarization, and its intensity I_d depends on the property of media, and also on the magnitude of the electrical field. Included in (6) the values of wave mismatch depend on the angle of tilt $\Delta\Theta$ of the wave vector of incident light from the direction of Bragg condition: $\Theta_B = \arcsin(\lambda_0/2\Lambda n)$, where Λ is the ultrasonic wavelength. In the case of light propagation in the direction, for which the projection of wave vector of incident light wave to a wave vector of ultrasonic wave is negative

$$\Delta k^+ = \Omega n/c + K\Delta\Theta. \qquad (7a)$$

For an opposite direction of the light propagation the acoustooptical interaction in gyrotropic media in the presence of external electrical field is also described by (4), where it is necessary to execute replacement in the expression for wave mismatch $\mathbf{K} \to -\mathbf{K}$ and in

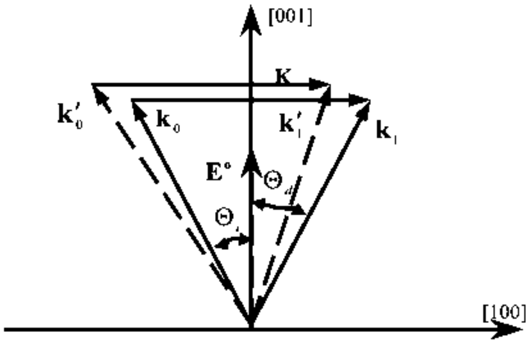

Fig. 1. Geometry of interaction (wave propagate in a forward direction): $\mathbf{k}_{0,1}$ and $\mathbf{k}'_{0,1}$ are wave vectors of incident (0) and diffracted (1) waves in the case of light propagating at Bragg angle and in the presence of a small mismatch $\Delta\Theta$.

the expression for a tensor κ_{ij}^{el}: $\mathbf{E}^0 \to -\mathbf{E}^0$. Then the field of light wave diffracting on transversal sound wave with a displacement vector, collinear to *y* axis, is described, as well as in the previous case, by the expression (6), in which the wave mismatch (we shall denote it with index "-") is determined by a ratio

$$\Delta k^- = -\Omega n/c + K\Delta\Theta\,, \qquad (7b)$$

and the value κ^{el} changes the sign on opposite one.

The power transmittance of an acoustooptical cell $T^{\pm}$ can be obtain from a ratio

$$T^{\pm} = 1 - \frac{\left|\mathbf{E}_1^{\pm}(l)\right|^2}{\left|\mathbf{E}_0^{\pm}(0)\right|^2} = 1 - \frac{I_d^{\pm}}{I^0}. \qquad (8)$$

Because of the effect difference of wave mismatch $\Delta k^{\pm}$ conditioned by Doppler included in expression for $I_d^{\pm}$, propagation of light waves on acoustooptical element in direct and opposite direction arises. The value of amplitude non-reciprocity is determined by the difference $\Delta I = T^- - T^+$. As it follows from (8), (6), the amplitude non-reciprocity depends on the magnitude of electroinduced anisotropy and on the gyrotropy of the crystal. This fact is demonstrated in Fig. 2 for a case of incidence a linear polarized light wave on the crystal $Bi_{12}SiO_{20}$. On calculation it was taken $\rho = 21.68^o\,/\,\mathrm{mm}^2$, $\lambda_0 = 0.6328\,\mu m$, $f = \Omega/2\pi = 200\,\mathrm{MHz}$, $I_a = 1\,\mathrm{W/cm}^2$, $l = 2$ cm. It is clear from Fig. 2, that accounting of gyrotropy leads to the essential change of $\Delta I(\Delta\Theta)$ dependence: instead of one central clearly expressed maximum (for a case, when on calculation the gyrotropy was neglected) there are two lateral maximums, the angular distance between which is determined by the value of parameter of gyrotropy. This feature is explained by the existence of two circularly polarized eigen waves propagated in the same direction in a gyrotropic cubic crystal. Applying an external electrical field causes the displacement and increasing of maximum of non-reciprocity. That is the result of electroinduced change of phase velocities and polarization of eigen light waves in the crystal. Let's mark that the neglecting of gyrotropic properties of media leads to incorrect determination of not only the position but also of the value of amplitude non-reciprocity maximum. On switching the direction of the applied electric field, as it is evident from Fig. 2, it is possible to change the sign of non-reciprocity and, therefore, to change the directions of lasing of the circular ring laser. As follows from (6), (8), polarization characteristics of incident light influences on the value of amplitude non-reciprocity (see Fig. 3, 4).

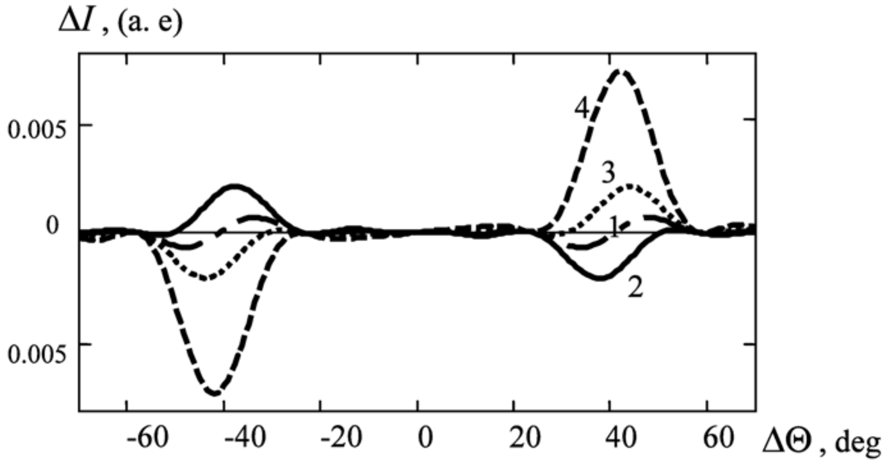

Fig. 2. Dependence of amplitude non-reciprocity on angular mismatch $\Delta\Theta$ at light diffraction on a ultrasonic wave in crystal $Bi_{12}SiO_{20}$ (incident light is linear polarized τ=0, χ=0) $E_z^0 = 0$ kV/cm (1), $E_z^0 = -1$ kV/sm (2), $E_z^0 = 1$ kV/sm (3), $E_z^0 = 4$ kV/sm (4).

As it is possible to show from (6), (8) the waves, passed through AO cell have different ellipticity and different direction of rotation of ellipse of polarization. Polarization non-reciprocity renders considerable influence on the efficiency of AO interaction and, therefore, on amplitude non-reciprocity.

By using the ratio (6), it is possible to analyze also the phase non-reciprocity $\Delta\Psi = \Psi^- - \Psi^+$, caused by difference of phases of waves, passed through the crystal. It follows from the calculation, the existance of the gyrotropy caused a doubling of phase non-reciprocity maximum on dependence $\Delta\Psi(\Delta\Theta)$. The principle meaning of the application of an electric field consists in the change of orientation and magnitude of non-reciprocity maximum. The last take place at values of angle $\Delta\Theta$, for which $\Delta I = 0$.

Conclusion

Interrelated non-reciprocity arises on the diffraction of light waves on traveling ultrasonic wave in gyrotropic cubic crystals with electroinduced

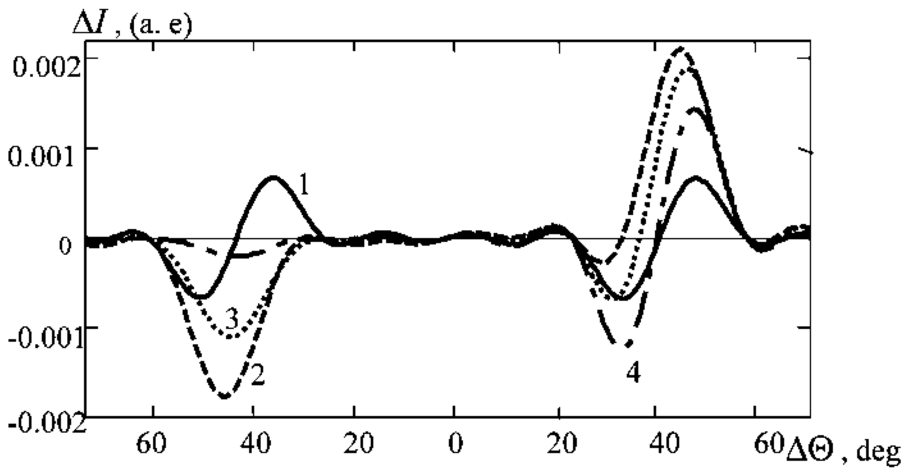

Fig. 3. Dependence of amplitude non-reciprocity on angular mismatch $\Delta\Theta$ at light diffraction on a ultrasonic wave in crystal $Bi_{12}SiO_{20}$ at different values of ellipticity of incident light polarization: τ=0 (1), τ=0.2 (2), τ= 0.5(3), τ=0.9 (4), $E_z^0 = 1$ kV/cm.

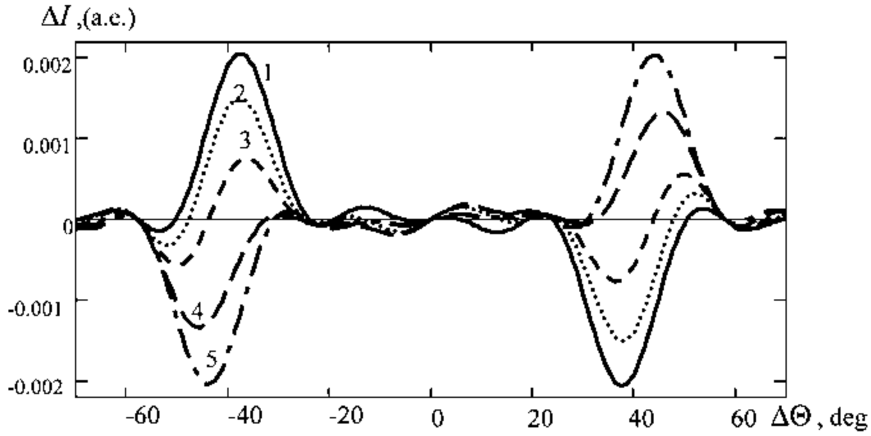

Fig. 4. Dependence of amplitude non-reciprocity on angular mismatch $\Delta\Theta$ at light diffraction on an ultrasonic wave in crystal $Bi_{12}SiO_{20}$ at different values of azimuth of incident light polarization: $E_z^0 = 1$ kV/cm, τ=0, χ=π/2 (1), χ=π/3 (2), χ=π/4 (3), χ=π/8 (4), χ=0 (5).

anisotropy amplitude, phase and polarization. The application of the external electrical field could permit to control the lasing of circular ring laser, obtaining single-wavelength or two-frequency generation without the change of orientation of a crystal, switching of direction and frequency of generation. The obtained results may be used for creation and optimization of parameters of non- reciprocal elements based on cubic gyrotropic crystals with electric and polarization controlling.

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