Crystallooptics of the circularly polarized waves

R.Vlokh

Institute of Physical Optics, Laboratory of the gradient optics and polarimetry, 23 Dragomanov Str., 79005, Lviv, Ukraine, vlokh@ifo.lviv.ua

Received 09.08.2001

Abstract

Article is devoted to the analysis and experimental study of the interaction between the circularly polarized waves as the eigen waves of gyrotropic crystals. The new effects such as two-beam refraction of the circular waves, acoustogyration diffraction of light, interferencion of the circularly polarized waves are phenomenologycaly predicted and experimentally studied. The phenomenological analysis and experimental study of the different types of optical activity that appear as the result of interaction of circular waves at the influence of electrical and magnetic field as well as mechanical strain is presented. The relations for the electro-, piezooptical and electro-, piezogyration effects in paraphase of the ferroelectrics-ferroelastics are obtained. It was shown that crystallooptics of the circularly polarized waves could be considered as the separate branch of traditional crystallooptics.

PACS: 42.81.Gs, 78.20.Ek, 35.55.-b

Contents

1.	Introduction	107
2.	Propagation of the circularly polarized waves in the gyrotropic media	108
	2.1.Surfaces of the refractive indexes, phase and beam velocities of the circularly polarized waves in the gyrotropic crystals on the wavelength of the isotropy point	108
	2.2.Effect of two-beam refraction	109
	2.3.Interference of the circularly polarized waves in the gyrotropic crystals.	113
	Conoscopic patterns	
3.	Parametrical crystallooptical effects at the presence of the circularly polarized	
	eigen waves	
	3.1.Change of the gyration surfaces at the electrogyration and piezogyration in the enantiomorphic crystals	116
	3.2.Acoustogyration diffraction of the light	118
	3.3. Symmetry aspects of the combined effects of the optical activity	121
	3.4.Relations for the electro-, piezooptical and electro-, piezogyration effects in the	122
	paraphase of the ferroelectrics-ferroelastics.	
4.	Conclusions	124
Re	ferences	124

1.Introduction

As it is well known the existing of the anisotropy in media, particularly, leads to the appearance of the optical birefringence. In this

case eigen waves possess linear polarization and different phase and group velocities. But generally if in anizotropic media, for example in crystals can exist spatial or frequency dispersion or thermodynamic flows the eigen waves should possess elliptical polarization. The frequency dispersion could be connected with the dissipation of optical energy as well as spatial dispersion with the gyration or non-homogeneity of media. The presence of flows of the charges or heat can induce the Faraday type optical activity (elliptically of eigen waves). The existing of the elliptically eigen waves in this three principle cases (frequency dispersion (1), spatial dispersion (2) and existing of flows (3)) can be demonstrated on the base of the Maxwell equation for the non-magnetic media

$$D^{\omega}_{i} = \varepsilon^{o}_{ij} E^{\omega}_{j} + \varepsilon''_{ij} \partial E^{\omega}_{j} / \partial t,$$

$$\varepsilon_{ij} = \varepsilon^{o}_{ij} + i\varepsilon''_{ij} \omega \qquad (1)$$

$$D^{\omega}_{i} = \varepsilon^{o}_{ij} E^{\omega}_{j} + \gamma_{ijk} \partial E^{\omega}_{j} / \partial x_{k}$$

$$\varepsilon_{ij} = \varepsilon^{o}_{ij} + i\gamma_{ijk} k_{k} \qquad (2)$$

$$D^{\omega}_{i} = \varepsilon^{o}_{ij} E^{\omega}_{j} + i\delta_{ijk} H_{k} E^{\omega}_{j}$$

$$\varepsilon_{ij} = \varepsilon^{o}_{ij} + i\delta_{ijk} H_{k}, \qquad (3)$$

(in the last relations flows of charges that induced magnetic fields is taking into account); where ε^{ρ}_{ij} and ε''_{ij} - real and imaginary part of the dielectric permittivity, D_i^{ω} and E_i^{ω} - electrical induction and strength of electric field on the optical frequency, H_k - strength of magnetic field, δ_{ijk} and γ_{ijk} axial and polar antisymmetric third rank tensors, k_k - wave vector x_k - coordinate and t - time. From the relations (1) - (3) it is visible that the existing of the frequency and spatial dispersion as well as flows of charges (due to the symmetry of the magnetic field and Onzager principle) lead to the appearing of the imaginary part in dielectric permitivity. From other side of view from the Ermit principle followed that the imaginary part of the dielectric permitivity should be only antisymmetrical. Antisymmetric part of the second rank polar tensor is dual to the axial vector ρ_n . It means that the vector of the electrical induction

$$D^{\omega}_{i} = [\rho_{n} \times E^{\omega}_{i}], \tag{4}$$

should turn in such media and elliptically eigen waves will exist.

According to the superposition principle total birefringence Δn consists from the linear Δn_l and circular birefringence Δn_c

$$\Delta n = (\Delta n_c^2 + \Delta n_l^2)^{1/2}.$$
 (5)

It means that two limit cases can exist: the presence of only linear birefringence and linear polarized eigen waves and the presence of only circular birefringence and circularly polarized eigen waves. The second case corresponds to the propagation of the optical beam along optical axes in gyrotropic crystal, propagation of optical beam in isotropic optically active media and propagation of the optical ray on the wavelength of the isotropic point in anizotropic crystal. In the last situation it is necessary to take into account the anisotropy of velocities of circularly polarized waves (left and right) and it seems reasonable to consider this case in more details.

The present paper is devoted to phenomenological analysis of propagation of circularly polarized waves in anizotropic, gyrotropic crystals and to the experimental investigations of effects that are connected with this.

2. Propagation of the circularly polarized waves in the gyrotropic media

2.1. Surfaces of the refractive indexes, phase and beam velocities of the circularly polarized waves in the gyrotropic crystals on the wavelength of the isotropy point

In the case when gyrotropic crystals are cubic or possess the isotropic point on some wavelength it is possible to analyze surfaces of the refractive indexes and phase and beam velocities of the circularly polarized waves [1,2]. For example for the linearly polarized eigen waves the surfaces of the phase and group velocities as well as refractive indexes are well known [3], they are mathematically coupled and reflect the relation between direction of the beam propagation and normal direction to the wave front. In general, in the anizotropical media the phase v and group u

velocities are different and the angle between them is equal

$$\psi = \arccos(v/u).$$
 (6)

The relation between ray refractive indexes (q=u/c) and wave refractive indexes can be written as

$$q = (n\cos\psi)^{-1}. (7)$$

In the isotropic media ψ =0, since there is no difference between phase and group velocities of the linearly polarized eigen waves. However, if the gyrotropic crystal possesses the point of the inversion of the linear birefringence at some wavelength then the existing of the circular birefringence

$$\Delta n_c = G/\overline{n}, \tag{8}$$

where G - is the scalar gyration parameter and n - is the average refractive indexes and should lead to the difference in v and u for the circularly polarized waves.

Let us construct the surfaces of phase and group velocities as well as refractive indexes

$$n_{l,r} = n_o \pm \Delta n_c/2,$$
 (9)

where n_o is the ordinary refractive indexes) of the left and right circularly polarized waves. Thus for example for the cubic gyrotropic crystals with the point group of symmetry 432 and 23 or gyrotropic liquids these surfaces should possess the spherical shape with different radii for the left and right waves. It is interesting to note that in the case of changing the sign the rotary power and crossing the refractive indexes n_l and n_r at some wavelength one can estimate the existing of the isotropic point for the circularly polarized waves. At the satisfaction of this condition in the media should propagate two circularly polarized eigen waves with the same velocities.

As well as gyration surfaces in the non cubic crystals possess the shape of the ovaloids then respective double surfaces of the refractive indexes and phase velocities also will be ovaloidal (it follows from the eq's (8), (9)) (Table 1)

However the surfaces of the group velocities and beam refractive indexes have to be ellipsoidal. The construction of the surfaces of the group velocities and beam refractive indexes it is possible to make geometrically on the base of respective known surfaces of group velocities and refractive indexes if to follow the procedure described in [4].

As it is visible from Table.1 the cone of the wave and the Poyngtings vectors directed from the center of ovaloids and ellipsoids to the points of the crossing of the surfaces is the cone of the optical axes of the circular eigen waves. In the axial symmetry crystals of the lower syngonys this cone is transformed to the elliptical or ovaloidal cone with the ellipse or oval in the cross section, respectively, while in the crystals that belong to the $\frac{1}{4}$ 2m, $\frac{1}{4}$, mm2 and m point group of symmetry circular optical axes should belong to the two mutually perpendicular planes. However the circular optical axes can exist only in the case when the symmetrical diagonal gyration tensor have the components with different signs. The optical axes which appear as the result of the crossing of the surfaces of the wave velocities it is reasonable to define as the wave polyaxes as well as - group velocities - ray polyaxes.

2.2.Effect of two-beam refraction

The difference in the group and phase velocities of the linearly polarized eigen waves take place in the effect of the conical refraction of light. At the light propagation through optically biaxial crystall along one of the wave axes this ray will split on the infinity numbers of rays that possess linear polarization with different azimuth of polarization. Inside the crystal optical energy will propagate along directions that belong to the hollow cone - cone of the internal conical refraction [5-7]. Outside the crystal plate light rays make the hollow cylinder.

Table 1. Surfaces of the phase and group velocities of circularly polarized waves in gyrotropic crystals on the wavelength of isotropic point.

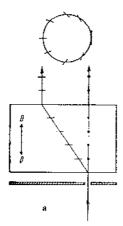
Point groups of symmetry	Gyration surfaces	Surfaces of the phase velocities	Surfaces of groups velocities
23, 432	, (N. N	Tu Ui Vi
622, 6 32, 3 422, 4	x V	Vr Vi Vi	Tu _l U _l
222, 2	x X	V _r V _r	V _Y V _Y
42m, 4, mm2, m		X X Y	Z U _j U _r

On the screen that is parallel to the crystal plate one can observe the ring diameter which do not change at the screen displacement while the polarization of the rays is changed to 180° around the ring. The effect of the external conical refraction is similar. At this effect unique direction of the ray axis corresponds infinity numbers of optical waves (Fig.1,b). It is interesting to remember that the finding the conical refraction was the result of the V.Hamilton scientific prediction (1832) which was made on the base of the wave surfaces, constructed by Fresnell. One year latter Lloyd experimentally confirmed this aragonite crystals [8]. The angle of the conical refraction for the major part of materials is quite small (see for example [4]) and can be written as

$$tan\alpha = N_p N_g ((N_p^{-2} - N_m^{-2})(N_m^{-2} - N_g^{-2}))^{1/2}, (10)$$

where N_p , N_m and N_g are refractive indexes.

Effect of the conical refraction can influence the nonlinear optic phenomena's [9-16]. The presence of the natural [12] or induced optical activity [17] leads to the changing of the manifestation of conical refraction. The presence of the ellipticity of eigen waves lead to the loosing of the degeneration of the phase and group velocities of eigen waves and to the disappearing of the conical refraction. As a result in the gyrotropic crystals in the vicinity of the optical axis two or four waves with different velocities can be propagated in the same



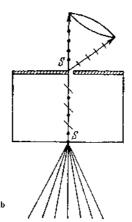


Figure 1. Propagation of rays at the internal (a) and external (b) conical refrection; OO - direction of the optical axis, SS - direction of the ray axis. Dots and dashes represent the directions of the oacilations of *E* vectors.

direction At the disappearing of the linear birefringence the problem of the conical refraction also disappears.

However as it was mentioned above in the gyrotropic crystals at the wavelength of the isotropic point circular optical polyaxis should exist [18, 19]. Since one could expect the appearing of the effect analogycal to the conical refraction due to the anisotropy and velocities difference of the circularly eigen waves. On the Fig.2 the cross section of the surfaces of the wave v and group u velocities of the circular waves at the wavelength of isotropic point is presented. The existing of the optical ray polyaxes - OR and wave polyaxes - ON lead to the possibility of the appearing of the effect of the two-beam refraction of the circularly polarized waves (Fig.2,3). On the base of geometrical consideration one can obtain the equations for the angle of internal two beam refraction

$$tan\alpha = \frac{(v_{rz}^{2}v_{ly}^{2} - v_{ry}^{2}v_{lz}^{2})\sqrt{(v_{ly}^{2} - v_{ry}^{2})(v_{rz}^{2} - v_{lz}^{2})}}{v_{ry}^{2}v_{ly}^{2}(v_{rz}^{2} - v_{lz}^{2}) + v_{rz}^{2}v_{lz}^{2}(v_{ly}^{2} - v_{ry}^{2})},$$
or
$$tan\alpha = \frac{8n_{o}^{2}(4n_{o}^{4} + Gg_{33})\sqrt{Gg_{33}}}{(4n_{o}^{4} - G^{2})(4n_{o}^{4} - g_{33}^{2})}$$

where n_o - is the refractive index at the wavelength of the isotropic point, g_{33} - component of the gyration tensor, $G=g_{11}cos^2\phi+g_{22}sin^2\phi$ -

scalar gyration parameter, ϕ - angle between X-axis and projection of the ray optical axis on the XY - plane, v_{rz} , v_{ry} , v_{lz} and v_{ly} - velocities of the left and right circularly polarized waves propagated along Z and Y axes. The ray passing at the external two beam refraction is presented on Fig.3.

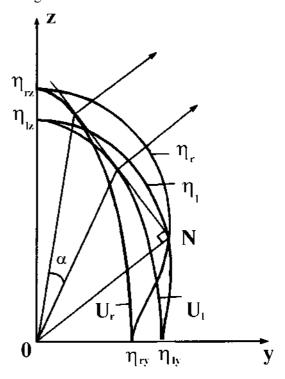


Figure 2. Crossection of the surfaces of group and phase velocities of circularly polarized waves and rays propagated in the internal two-beam refraction.

Relation for the external two beam refraction can be written

$$tan\beta = \frac{(v_{rz}^2 v_{ly}^2 - v_{ry}^2 v_{lz}^2) \sqrt{(v_{ly}^2 - v_{ry}^2)(v_{rz}^2 - v_{lz}^2)}}{v_{rz} v_{lz} v_{ry} v_{ly} (v_{rz}^2 - v_{lz}^2 + v_{ly}^2 - v_{ry}^2)},$$
or
$$tan\beta = \frac{8n_o^2 (4n_o^4 + Gg_{33})(G + g_{33}) \sqrt{Gg_{33}}}{g_{33} (4n_o^4 - G^2)^2 + G(4n_o^4 - g_{33}^2)^2}.$$

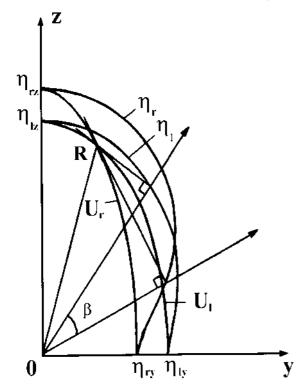


Figure 3. Crossection of the surfaces of group and phase velocities of circularly polarized waves and rays propagated in the external two-beam refraction.

In the crystals that belong to the point groups of symmetry $\overline{4}2$ m, mm2 and m angles of the external and internal two beam refraction are the same and relations (11) and (12) in the condition $g_{33}=g_{11}=G$ can be rewritten as

$$tan\alpha = tan\beta = \frac{8n_o^2 g_{11} (4n_o^4 + g_{11}^2)}{(4n_o^4 - g_{11}^2)^2}.$$
 (13)

The estimation of the angle of the external and internal two beam refraction one can make for the AgGaS₂ crystals that belong to the $\overline{42}$ m point group of symmetry and possess isotropic point at the wavelength λ =498nm (g_{II} =4×10⁻³, n_o =2.7) [20-22]. This angle (α = β =3.77') is quite small and the experimental finding of two beam refraction in usual crystals can be a nontrivial problem.

As well as two conjugate beams correspond to one direction of the phase optical axes of the circular waves (internal refraction) out of the crystal this beam should be parallel (Fig.4,a). The polarization of this beam will be circular with the different directions of the rotation of vector E. External refraction could be observed at the light propagation along the one of ray optical axes. The manifestation of this effect is as: if two circular beam incidents on the crystal face and angle between them equals to β they will propagate in the parallel directions inside the crystal and outside the crystal the angle between them will equal to β (Fig.4,b).

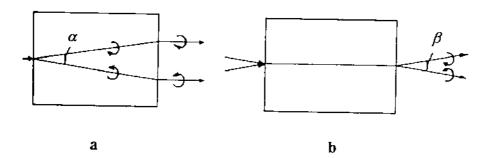


Figure 4. Schematical propagation of the rays in internal (a) and external (b) two-beam refraction of circular waves (axial vectors present the directions of the rotation of vector *E* of the circularly polarized waves).

2.3.Interference of the circularly-polarized waves in the gyrotropic crystals. Conoscopic patterns

The interference patterns that appear at the propagation of the focused beam through anisotropical crystals which are between the crossed polarizors possess the information about the birefringence of these crystals. It is known [23] that the existance of the optical activity leads to the changing in the conoscopic figures. However in the gyrotropic crystals on the wavelength of the isotropic point one can observe the interference patterns only due to the existance of the circular birefringence.

To analyze the conoscopic patterns one can use the approach described in [24]. Let us consider the linearly polarized light beam that after polarizer propagate through flat parallel crystal plate which is prepared of the gyrotropic crystal. The emergent from the crystal waves possess the phase difference

$$\delta = \frac{2\pi h}{\lambda \cos \theta} (\Delta n_l^2 + \Delta n_c^2)^{1/2}, \tag{14}$$

where θ - refractive angle inside the crystal, h - thickness of the plate, $h/\lambda\cos\theta$ - average geometrical way of optical beam in crystal, Δn_l - linear birefringence, Δn_c - circular birefringence, λ - wavelength. If the optical radiation with the wavelength of isotropic point propagates through the crystals (14) can be rewritten as

$$\delta = 2\pi h \Delta n_c / \lambda \tag{15}$$

If to take into account that $\Delta n_c = G/n$ then phase difference of the circularly polarized waves will be written

$$\delta = 2\pi h \Delta G / \lambda n \tag{16}$$

By the construction of the surfaces of the constant phase difference $\delta(h,\theta)$ =const and by making their cross sections by the plane h=const one can obtain the izochromates.

For example in the crystals which belong to the point groups of symmetry 622, 6, 422, 4, 32, 3 the gyration surface should possess the shape of the two sign ovaloid with gyration tensor components $-g_{II} = -g_{22} \neq g_{33}$. In the spherical coordinate system gyration of these crystals can be written as

$$G = g_{33}\cos^2\theta - g_{11}\sin^2\theta. \tag{17}$$

By introducing (17) to (16) we can obtain the relation of the constant phase difference $\rho(g_{33}cos^2\theta-g_{11}sin^2\theta)=C$, C=const. (18)

Let us return to the Cartesian coordinate system

$$\rho^{2} = x^{2} + y^{2} + z^{2},$$

$$\rho^{2} \sin^{2} \theta = x^{2} + y^{2},$$

$$\rho^{2} \cos^{2} \theta = z^{2}$$
(19)

and accordingly to (18) the surfaces of the constant phase difference should be determined by the equation

$$(g_{11}(x^2+y^2)-g_{33}z^2)^2=C^2(x^2+y^2+z^2).$$
 (20)

These surfaces can be obtained at the rotation of the curves

$$(g_{11}x^2-g_{33}z^2)^2=C^2(x^2+z^2).$$
 (21)

around z axis (such typical curve is presented on Fig.5)

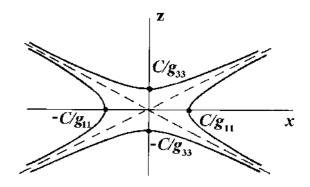


Figure 5. Crossection of the constant phase difference surface by the plane Y=0 for the axial crystals of middle syngonys.

The asymptotic path of these curves $z=\pm x(g_{11}/g_{33})^{1/2}$ (22)

coincides with the directions in which phase difference between two circular waves equal zero, i.e. circular optical axes [25,26]. So, by using the surfaces (20) cross sections by the plane that is perpendicular to the direction of light propagation one can construct the conoscopic patterns for all groups of symmetry (Table 2).

Crossections of the constant phase difference surfaces by the M - plane MLz 2 > C Crossection of the constant phase difference surfaces by the principle crystallophysical planes. $M \perp y$ MLX x = 0Equation of the constant $(g_{11}x^2 + g_{22}y^2)^2 - 2g_{11}g_{33}x^2z^2$ phase difference surfaces $(g_{11}(x^2 + y^2) + g_{33}z^2)^2 = C(x^2 + y^2 + z^2)$ $(g_{11}(x^2 + y^2) - g_{33}z^2)^2 = C(x^2 + y^2 + z^2)$ $2g_{22}g_{33}y^2z^2 + g_{33}z^4 =$ $z^4 = C^2(x^2 \! + y^2 + z^2)$ Symmetry of | Gyration surfaces 222, 2, 1 622, 6, 422, 4 622, 6, 32, 3, 422, 4 622, 6, 422, 4 32, 3, 32, 3, crystals

Table 2

Table 2		ı	1		ı	ſ
Tal accs by the M – plane	M⊥z	v ← 0 = z	0 = z	$\sum_{x=0}^{y}$	$\begin{array}{c c} x \\ \hline \\ z = 0 \end{array}$	z = 0
ant phase difference surf	M⊥y	$\begin{array}{c c} x & \sqrt{0 = h} \\ \hline \\ x & \sqrt{1 + h} \end{array}$	$\begin{array}{c c} x & \sqrt{g = h} \\ \hline \\ x & - \\ \end{array}$	$y = C\sqrt{2}$	z = 0	$ \begin{array}{c c} x \\ \hline \\ \lambda = 0 \end{array} $
T Crossections of the constant phase difference surfaces by the M – plane	M⊥x	$\begin{array}{c c} & & & \\ \hline \\ \hline$	$\begin{array}{c c} & & & \\ \hline &$	$X = C\sqrt{2}$	$Z \longrightarrow A \longrightarrow $	$\begin{array}{c c} & y & x \\ \hline & z = 0 \end{array}$
Equation of the constant phase difference surfaces		$(x^2+y^2)^2 = C^2(x^2+y^2+z^2)$	$(x^2 + y^2)^2 = C^2(x^2 + y^2 + z^2)$	$x^2y^2 = C^2(x^2 + y^2 + z^2)$	$(g_{11}x^2 - g_{33}z^2)^2 = C^2(x^2 + y^2 + z^2)$	$(g_{11}x^2 + g_{22}y^2)^2 = C^2(x^2 + y^2 + z^2)$
Symmetry of Gyration surfaces crystals		,	y Service of the serv	A S	- 4	- O ×
Symmetry of crystals		622, 6, 2, 3, 422, 2	42m, 4	mm2, m	222, 1	222, 1

From the obtained conoscopic patterns one can make following conclusions:

1.In the middle syngony crystals of the axial symmetry groups with different signs of the diagonal components of the gyration tensor at the light propagation in the z direction conoscopic pictures should possess the view of rings as well as in the directions of x and y crystallophysical axes they will be similar to the usual conoscopic patterns that can be observed in linearly birefringence crystals in the directions of the principal crystallophysical axes.

2.In the crystals that belong to the $\overline{4}2m$, $\overline{4}$, mm2 and m point group of symmetry conoscopic pictures that could be observed at the light propagation along the principal crystallophysical axes will be similar to the usual conoscopic patterns that can be observed in linearly birefringence crystals in the directions of the principal crystallophysical axes.

3.In the middle syngony crystals of the axial symmetry groups with the same sing of the diagonal components of the gyration tensor at the light propagation in the x and y directions conoscopic patterns should possess the view of ellipses.

4. In the lower syngony crystals of the axial symmetry groups at the light propagation in the principal directions conoscopic patterns should possess the view of ovaloids.

3. Parametrical crystallooptical effects at the presence of the circularly-polarized eigen waves

3.1. Change of the gyration surfaces at the electrogyration and piezogyration in the enantiomorphic crystals

The enantiomorphism in crystals that belong to 1, 2, 3, 4, 6, 222, 32, 422, 622, 23 and 432 point group of symmetry leads to the difference in the sign of the gyration tensor at the changing of the sign of the coordinate system. In this case the behavior of the electro- and piezogyration effect which can be described by the relations

$$\Delta g_{ij} = \gamma_{ijk} + \delta_{ijkl} E_k E_l + \beta_{ijkl} \sigma_{kl}, \qquad (23)$$

where γ_{ijk} δ_{ijkl} , β_{ijkl} - third and forth rank axial tensors, will be defined by the change of properties of the tensors γ_{ijk} , δ_{ijkl} , β_{ijkl} at the changing of the sign of coordinate system.

The change of the sign of the enantiomorphism does not change the linear electrogyration tensor γ_{ijk} i.e. does not lead to the change of sign of the increment of the gyration tensor at the same direction of the electric field vector. In this case in the enantiomorphic crystals the application of the same electric field should lead to different changing of the gyration surfaces (Fig.6).

The axial fourth rank tensor that describs quadratic electrogyration and piezogyration at the transmission from the left to the right

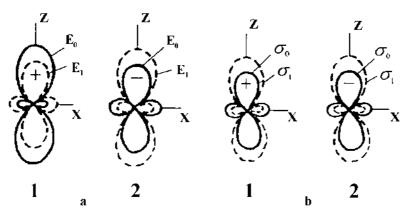


Figure 6. Change of the gyration surfaces for the right (1) and left (2) crystals at linear electrogyration (a) and piezogyration (b).

coordinate system will change the sign to the opposite. Then the increment of the gyration tensor will also change the sign. In this case the mechanical stress and the electric field of the same sign applied to the enantiomorphic crystals will lead to the same deformation of the gyration surface. In Table 3 the relations that describe induced gyration in crystals with different signs of enantiomorphism are presented.

Table 3. The relations that describe induced gyration in crystals with different sign of enantiomorphism

Right crystals	Left crystals
$\Delta g_{ij} = \gamma_{ijk} \; E_k$	$\Delta g_{ij} = \gamma_{ijk} \; E_k$
- Δg_{ij} = - $\gamma_{ijk} E_k$	$-\Delta g_{ij} = \gamma_{ijk} \; E_k$
$\Delta g_{ij} = \delta_{ijkl} E_k E_l$	- Δg_{ij} = - $\delta_{ijkl} E_k E_l$
- Δg_{ij} = - $\delta_{ijkl} E_k E_l$	$\Delta g_{ij} = \delta_{ijkl} E_k E_l$
$\Delta g_{ij} = \beta_{ijkl} \delta_{kl}$	$-\Delta g_{ij} = \beta_{ijkl} \delta_{kl}$
$-\Delta g_{ij} = - \beta_{ijkl} \delta_{kl}$	$\Delta g_{ij} = \beta_{ijkl} \delta_{kl}$

The piezogyration effect we studied in the enantiomorphic quartz crystals at the light (λ =632.8nm) propagation along z axis and compressive stress σ_{33} by the measuring of polarization plane rotation [27]. The dependencies are presented on Fig.7.

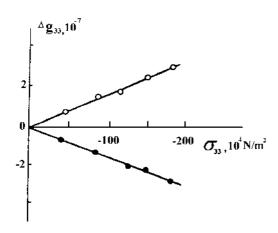


Figure 7. The dependencies of the changes of gyration tensor component on the compressive stress σ_{33} in enantiomorphic (\bullet - right, o - left) quartz crystals at the light (λ =632.8nm) propagation along z axis.

The values of the piezogyration tensor components are calculated as $\beta_{3333}=\pm(1.5\pm0.3)\times10^{-13}\text{m}^2/\text{N}$ ("-" for right quartz, "+" for left quartz).

The quadratic electrogyration effect for the first time was observed in quartz crystals [28] and then studied in KDP crystals [29]. However in both cases the measurements were held on the condition of presence of the linear birefringence. In the pure case quadratic electrogyration could be observed in crystals that belong to 32, 422, 622 point groups of the symmetry at the application of the electric field and light propagation along z axis. The change of the gyration tensor can be written as

$$\Delta g_{33} = \delta_{3333} E_3 E_3 \tag{24}$$

The investigations of the quadratic electrogyration was held by using the measuring of the polarization plane rotation in TeO₂ crystals at the light propagation (λ =632.8nm) and electric field application along z axis. At Fig.8a,b [30] the dependence of the specific rotation power on the square electric field is presented. The component of the electrogyration tensor calculated from this dependence equals $\delta_{3333} = \Delta\rho\lambda n_o/\pi E_3^2 = (6.6\pm1.7)\times10^{-20} \text{m}^2/\text{V}^2$ is of the same order as it is in the quartz crystals [28].

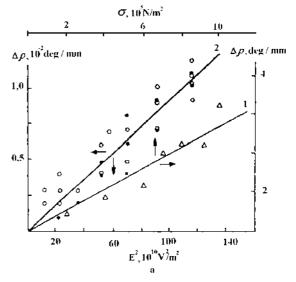


Figure 8a. Dependence of the change of the gyration tensor component in the TeO₂ crystals on mechanical stress (1) and electrical field (2);

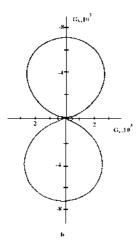


Figure 8b. View of the gyration surface in TeO_2 crystals (λ =632.8nm, T=295K)

The application of the uniaxial stress to the TeO_2 crystals along the direction perpendicular to the optical axis leads to the optical byaxiality of these crystals due to the piezooptical effect. More over the increasing of the stress will lead to the increasing of the angle between the optical axes. Since, if to study the optical activity along one of the optical axes by the measuring of the polarization plane rotation in different angles between optical axes and by solving the system of the equations

$$\rho_1 \lambda n/\pi = g_{33} cos^2 V_1 + g_{11} sin^2 V_1$$
, $\rho_2 \lambda n/\pi = g_{33} cos^2 V_2 + g_{11} sin^2 V_2$, (25) where $\rho_I = 1139 \text{rad/m}$ at $V_I = 5^\circ$, $\rho_2 = 1115.2 \text{rad/m}$ at $V_2 = 5.5^\circ$ and $\rho_3 = 921.1 \text{rad/m}$ at $V_3 = 8^\circ$ (V - is the angle between optical axis and z axis) one can calculate the gyration surface for the TeO₂ crystals. As it was found the $g_{II} = (-7.1 \pm 0.8) \times 10^{-3}$ is on the order of magnitude smaller than $g_{33} = (6.3 \pm 0.5) \times 10^{-4}$ and possesses other sign (Fig.8).

3.2.Acoustogyration diffraction of the light

The possibility of the light diffraction on the periodical distribution of the imaginary part of the dielectric permitivity was shown in [30-33]. In this case photorefractive grating was caused by the electrogyration effect. As the result the authors of [30] determined the electrogyration

constant (γ_{41} =(1.3±0.26)×10⁻⁹) for the Bi₁₂TiO₂₀ crystals.

From other side of view at the acoustooptical diffraction the light beam is deviated by the grating of the real part of the dielectric permitivity induced through elastooptical effect. But if to take into account the elastogyration effect which could be described by the equation

$$\Delta g_{ij} = \beta_{ijkl} X_{kl}, \qquad (26)$$

where β_{ijkl} - is the forth rank axial tensor and X_{kl} - is the deformation tensor. The value that characterizes the optical energy transferring into J - diffraction order can be written as [34]

$$q_i = k_i^2 \Delta n / k_{ix} n = 2\pi \Delta n / \lambda \cos \theta_o$$
, (27)

where k_j - is the wave vector of diffracted beam, k_{jx} - is the projection of the wave vector on the x - axes (in this case optical beam is propagated at θ_o - angle to the x axes of crystal). Then if to take into account the superposition principle of the linear and circular birefringence $\Delta n = (\Delta n_l^2 + \Delta n_c^2)^{1/2}$ and by using the relation of the deformation and the acoustic power P_a one can rewrite (27) as

$$q_j = \pi (P_a(M_2 + M_2')/lb)/\lambda \cos \theta_o$$
, (28) where l and b are the cross selection dimensions of the acoustic beam, M_2 and $M_2' = \beta^2_{ijkl}/n^2 \rho v^3$ are coefficients of the acoustooptical and acoustogyration quality, respectively, ρ - crystal density, v - sound velocity.

At the account of the elastogyration the relation for the electric field strength of the electromagnetic wave can be written as

$$E_{i}^{\omega \pm \Omega} = (a_{ij}^{o} + p_{ijkl} X_{kl}^{\Omega} + i e_{ijm} \beta_{mnk} l X_{kl}^{\Omega} k_{n}) D_{j}^{\omega},$$

$$\bar{\beta}_{ijnkl} = e_{ijm} \beta_{mnkl}, \qquad (29)$$

where ω and Ω - frequencies of the optical and acoustical waves, respectively.

Let us analyze the conditions on which acoustogyration diffraction of light could be observed in pure case with out acoustooptical one at collinear interaction. First condition: crystals should be noncentrosymmetrical point group of symmetry because tensor β_{mnkl} can be non zero only in such media. Secondly: the polarization

of the acoustic wave that propagates in the same direction as the incident and diffracted optical wave should be like that induced by this acoustic wave deformation must lead to the change of the gyration tensor. And finally elastogyration tensor should possess effective components in which *i*, *j*, *n* -indexes are different. From the latter condition follows that

acoustogyration diffraction of the light can be only anizotropical. [35]. In the Table 4 different geometrics of the experiment of the observation of the collinear acoustogyration light diffraction at the light and sound propagation along principle crystallophysical axis in the crystals that belong to the different point groups of symmetry are presented.

Table 4. Conditions of the observation of the collinear acoustogyration light diffraction.

Group of the symmetry	Direction of the propagation and polarization of acoustical wave	Deformation tensor components	Effective elastogyration coefficient
1	2	3	4
2, 2 Y	X, L	ϵ_{11}	β ₁₁₁₁
	X, S	$oldsymbol{arepsilon}_{13}$	β ₁₁₁₃
	Y, S	$oldsymbol{arepsilon}_{12}$	β_{2212}
	Y, L*	$oldsymbol{arepsilon}_{22}$	β_{2222}
	Y, S*	ϵ_{31}	β_{2231}
	Z, S	$oldsymbol{arepsilon}_{13}$	β_{3313}
	Z, L	ε ₃₃	β_{3333}
2, 2 Z	X, L	ϵ_{11}	β ₁₁₁₁
	X, S	$oldsymbol{arepsilon}_{12}$	β ₁₁₁₂
	Y, L	$oldsymbol{arepsilon}_{22}$	eta_{2222}
	Y, S	ϵ_{12}	β_{2212}
	Z, L*	ϵ_{33}	eta_{3333}
M, m⊥Y	X, S*	ϵ_{12}	β ₁₁₁₂
	Y, S	ε_{12} or ε_{23}	β_{2212} or β_{2223}
	Z, S*	ϵ_{23}	β_{3323}
M, m⊥Y	X, S*	ϵ_{13}	β ₁₁₁₁
	Z, S	ε_{13} or ε_{23}	β_{3313} or β_{3323}
	Z, S*	ϵ_{32}	β_{3332}
222	X, L	ϵ_{11}	β_{1111}
	Y, L	\mathbf{v}_{22}	β_{2222}
	Z, L	ϵ_{33}	β_{3333}
mm2	X, S	ϵ_{12}	β_{1112}
	Y, S	ϵ_{12}	β_{2221}
3	X, L*	ϵ_{11}	β_{1111}
	X, S*	ε_{12} or ε_{13}	β_{1112} or β_{1113}
	Y, S*	ε_{12} or ε_{13} or ε_{23}	β_{2212} or β_{2213} or β_{2222}
	Y, L*	ϵ_{22}	eta_{2222}
	Z, L	€33	β_{3333}
3m	X, S	ε_{12} or ε_{13}	β_{1112} or β_{1113}
	Y, S*	ε_{12} or ε_{13}	$\beta_{2212} \text{ or } \beta_{2213}$
32	X, L*	ϵ_{11}	β_{1111}
	Y, L	ϵ_{22}	eta_{2222}
	Y, S	$oldsymbol{arepsilon}_{23}$	β_{2223}
	Z, L	ϵ_{33}	β_{3333}

Group of the			Effective
symmetry	propagation and	tensor	elastogyration
	polarization of acoustical	components	coefficient
	wave		
4	X, L	ϵ_{11}	β_{1111}
	X, S	ϵ_{12}	β_{1112}
	Y, S	ε_{12} or ε_{23}	β_{2122} or β_{2222}
	Y, L	$oldsymbol{arepsilon}_{22}$	β_{2222}
	Z, L	€33	β_{3333}
$\overline{4}$	X, L	ϵ_{11}	β_{1111}
	X, S	ϵ_{12}	β_{1112}
	Y, S	ϵ_{12}	β_{2212}
	Y, L	$oldsymbol{arepsilon}_{22}$	β_{2222}
422	X, L	ε ₁₁	β ₁₁₁₁
	Y, L	$oldsymbol{arepsilon}_{22}$	β_{2222}
	Z, L	£ ₃₃	β_{3333}
	X, L	ε ₁₁	β ₁₁₁₁
12111	Y, L	ϵ_{22}	β_{2222}
4mm	X, S	ϵ_{12}	β_{1112}
	Y, S	$oldsymbol{arepsilon}_{12}$	β_{1112} β_{2212}
6	X, L		β ₁₁₁₁
	X, E X, S	ε ₁₁	
	Y, S	$oldsymbol{arepsilon}_{22}$	β_{1112}
	Y, L	ϵ_{12}	β_{2212}
	Z, L	$oldsymbol{arepsilon}_{22}$	eta_{2222}
(22		ϵ_{33}	β_{3333}
622	X, L	ϵ_{11}	β_{1111}
	Y, L	ϵ_{22}	β_{2222}
	Z, L	€33	β_{3333}
6	X, S*	ϵ_{13}	β_{1113}
	Y, S*	ϵ_{13}	β_{2213}
6m2	X, S	ϵ_{13}	β_{1113}
	Y, S	ϵ_{13}	β_{2213}
6mm	X, S	ϵ_{12}	β_{1112}
	Y, S	ϵ_{12}	β_{2212}
23	X, L	ϵ_{11}	β ₁₁₁₁
	Y, L	$oldsymbol{arepsilon}_{22}$	β_{2222}
	Z, L	£ ₃₃	β_{3333}
432	X, L	ε ₁₁	β ₁₁₁₁
	Y, L	$oldsymbol{arepsilon}_{22}$	β_{2222}
	Z, L	ϵ_{33}	β_{3333}
L	1 '	~33	دددد ۲

So, one of suitable possibilities of the separation of acoustooptical and acoustogyration light diffraction is collinear interaction at the propagation of the acoustical wave and optical waves along the three fold, forth fold or six fold symmetry axis of optically uniaxial or cubic gyrotropic crystals. In such geometry experiment the energy transferring between left and

right circularly polarized waves should be observed. Then in the result of action of the circular left and right waves with different intensities the emergent from crystal wave should possess elliptical polarization. This elliptical wave can be transformed into linear polarized wave by a quarter wave plate. The angle of rotation of azimuth $\Delta \varphi$ of this linear polarized

wave out of a quarter wave plate will be determined by the acoustogyration effectivity $\eta = tan\Delta\varphi$.

The study of the acoustogyration light diffraction was held on the enantiomorphic quartz crystals at the light and sound propagation along optical axis. The synchronism condition in this case is satisfied at a quite low frequency of the acoustic wave f_o =2MHz. The ceramic piezoelectric transducer that produced longitudinal acoustic wave is used in this experiment. The incident optical beam was propagated through the hole in the transducer. It is interesting to note that in the left and right quartz crystals diffracted beam possesses left and right circular polarization (Fig.9).

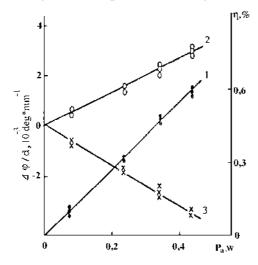


Figure 9.Dependence of the acoustogyration light efficiensy (1) and specific rotation of the polarization plane out of quater wave plate in the enantiomorphical spacimens of quartz crystals (2,3) on acoustic power (λ =632.8nm, T=295K).

This fact is connected with the difference in the synchronism conditions for right $(k_l+k_{ac}=k_r)$ and left $(k_r+k_{ac}=k_l)$ quartz crystals [36, 37].

3.3. Symmetry aspects of the combined effects of the optical activity

As it is known the two different types of optical activity differ from the symmetry (gyration - ∞ 2 and Faraday effect - ∞ /m) and by the experimental manifestation. Experimentally this

effect can be separated by the change of the sign of the wave vector, that leads to the doubling of the Faraday rotation and compensation of the gyration rotation. However observed optical rotation in the pyroelectric crystals under the application of the magnetic field [38] which was explained as magnetogyration [38, 39] and then by the combined influence of electrical and magnetic fields [40, 41] were also separated from the Faraday rotation by changing the sign of wave vector. In such case a reasonable is the question is: what is the symmetry of the found effect?

Let us analyze the reason of the manifestation of different effects of optical activity in more details [42-44]. As well as the Faraday effect consist exist in the inducing of the antisymmetrical part of dielectric permitivity with the symmetry ∞/m by the magnetic field the symmetry of this effect should be ∞/m . According to the relation (3) the sign of the rotation of polarization plane at the Faraday effect does not depend on the sign of wave vector but depend on the sign of the magnetic field. The gyration possesses the symmetry of the gyration tensor - ∞ 2 and according to (2) depends on the sign of the wave vector. A combined magnetoelectric optical activity that is described by the relation

$$\varepsilon_{ij} = \varepsilon_{ij}^{\ o} + ie_{ijl} \delta_{lkm} H_k E_m, \tag{30}$$

where E_m - is the strength of electrical field H_k - strength of magnetic field and δ_{lkm} - third rank polar tensor. The symmetry of this combined effect is determined by the symmetry of antisymmetrical part of dielectric permittivity - ∞ /m. This effect can be separated from the Faraday rotation by the change of the sign of electric field. Note that piezomagnetic optical activity ($\varepsilon_{ij} = \varepsilon_{ij}^{\ o} + i e_{ijl} Q_{lkmn} H_k \sigma_{mn}$) will possess the symmetry ∞ /m as well as piezoelectric optical activity ($\varepsilon_{ij} = \varepsilon_{ij}^{\ o} + i e_{ijl} (g_{lk} + \theta_{lkmnp} \sigma_{mn} E_p) k_k$ - ∞ 2.

From the moment of finding of the non trivial optical activity which appeared in magnetic field in the crystals LiJO₃, CdS and

Pb₅Ge₃O₁₁ its description was clarified few times. Firstly the difference in the polarization plane rotation angle at the opposite directions of wave vector was explained as magnetogyration effect. The difficulties of this explanation was connected with the limiting of the Onzager principle. Then this effect was explained as an optical activity induced by a combined influence of magnetic field and electrical polarization as well as it was observed only in pyroelectrics and ferroelectrics and was separated by the change of sign of polarization. However as it was mentioned above such effect should possess the symmetry ∞ /m and its separation by the change of sign of wave vector is non understandable. More over the appearance of this effect at the ferroelectric phase transition in Pb₅Ge₃O₁₁ crystals should be described on the basis of the group symmetry of paraelectric phase but not ferroelectric one. From this follows that effect that linearly depend on P_s should be forbidden by the symmetry of paraelectrical phase. From other side of view the explanation of this effect as magnetoelectrical (magnetopolarization) optical activity in the pyroelectrics also is non correct. The separation of this optical activity from the Faraday rotation was held by turning the sample by 180° around the axis that is perpendicular to the optical axis and in such a way the change of the sign of P vector. But in this operation the components of the tensor δ_{lkm} also should change the sign and rotary power which is proportional to the $\delta_{lkm}H_kP_m$ induced by the PH action can not be separated from the Faraday rotation. So in the case of pyroelectrics it is reasonable to return to the explanation of observed optical activity as magnetogyration as well as at the described operation magnetogyrational rotation of the polarization plane $\rho_l \sim \delta'_{lkm} H_k k_m$ will change the sign due to the change of sign of components of tensor δ_{lkm} . It is necessary to note that the Onzager principle can not be applied to the dissipative media while CdS pyroelectric crystals possess quite strong

interband absorption at the wavelength 632.8nm. It means that the magnetogyration in this crystals is not forbidden.

A similar situation was observed at switching of the enantiomorphic domain structure in the Pb₅Ge₃O₁₁ crystals. The additional rotation of the polarization plane that appears in magnetic field was separated from the Faraday effect the switching of domains. This additional effect can not be explained as magnetopolarization optical activity as well as under the switching simultaneously change the both sign - P_s and δ_{lkm} . But it can be explained as magnetogyration. The temperature dependence of the Faraday effect in the multidomain lead germanate crystals at the phase transition can be described as the magnetopolarization optical activity induced by quadratic spontaneous polarization $\rho_l \sim \chi_{lkmn} P_k P_m H_n$ (linear effect is forbidden by the symmetry of crystal in the paraelectric phase - 6). The temperature dependence of the magnetogyration in this case can be described as the effect induced by spontaneous polarization $\delta'_{lkm} = R_{lkmn}P_n$. It is interesting to note that in [38] effect induced by external fields of the magnetoelectrical optical activity in lead germanate crystals which was separated from Faraday rotation by the comparising of the rotary power at consequently applied electrical and magnetic fields was also observed.

Since, in [38-44] simultaneously was found as combined magnetoelectrical optical activity as well as the magnetogyration in dissipative media.

3.4.Relations for the electro-, piezooptical and electro-, piezogyration effects in the paraphase of the ferroelectrics-ferroelastics.

As it will be shown below for the existing of the coupling between effects of elastooptics, elastogyration, electrooptics and electrogyration following conditions should be satisfied:

1.Tensors of polarization constants - a_{ij} , gyration constants - g_{ij} , mechanical deformation -

 X_{mn} and electical field - E_k should be transformed by same irreducible representation of the point group of symmetry of a crystal.

2.Electrooptical tensor - r_{ijk} and electrogyration tensor - γ_{ijk} as well as elastooptical tensor p_{ijmn} and elastogyration tensor - β'_{ijmn} should possess the same view.

3. The crystal should be in the paraelastic-paraelectric phase and crystal should possess proper phase transition.

At the satisfaction of the first condition tensors a_{ij} and g_{ij} will be coupled with respective field values. Necessity of the existing of second condition is connected with the fact that external mechanical stress and electrical field should lead to the changes of the optical indicatrix and gyration surface of the same type. Finally, elastooptical, elastogyration, electrooptical and elec-

trogyration coefficients that described the change of refractive indexes and optical activity at ferroelectric-ferroelastic phase transition are the coefficients of paraelectric-paraelastic phase and it is indifferent how to describe this changes - or by electrooptical and electrogyration effect induced by spontaneous polarization or by elasto-optical and elastogyration effect induced by spontaneous deformation. So for this effects induced in paraelectric-paraelastic phase next relations should satisfied

$$\Delta a_{ij} = r_{ijk} E_k = p_{ijkl} X_{kl},$$

$$\Delta g_{ij} = \gamma_{ijk} E_k = \beta'_{iimn} X_{kl}.$$
(31)

From equations (31) one can obtain the relation

$$\beta'_{ijmn}d_{mnk}/p_{ijmn}d_{mnk} = \gamma_{ijk}/r_{ijk}, \qquad (32)$$

where d_{mnk} - is the tensor of electromechanical coupling. For the different point group of symmetry this relations are presented in Table 5.

Table.5. Relations for the electro-and elastooptical and electro- and elastogyration effects.

Change of the	Components	Relations for the electro-and elastooptical and electro- and
symmetry at	of spontaneous	elastogyration effects.
phase transition	polarization	
	vectors	
23 - 2	P_1, P_2, P_3	$\beta_{44}^{p}_{44} = \Gamma_{41}^{r}_{41}^{r}_{41}$
6 - 2	P_1, P_2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
32 – 2	P_1 P_2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
3 - 1	P ₁ , P ₂	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
422 - 2	P_1, P_2	β /p =γ /r 44 44 41 41
4 - 2	P_1, P_2	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
4 2m - 2	P_1, P_2	$\beta / p = \gamma / p$ $44 44 41 41$
4 - 1	$\begin{array}{c} P_1 \\ P_1 \\ P_2 \\ P_2 \end{array}$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
222 - 2	P_1	β /p =γ /r 44 44 41 41
222 - 2	P ₂	$ \beta /p = \gamma /r 55 55 55 52 52 \beta /p = \gamma /r $
	P ₃	66 66 63 63
2 - 1	P_1	β e +β e /p e +p e = Υ /r 44 41 45 51 54 41 45 51 41 41
2 z	\mathbf{P}_1	$ \beta e + \beta e / p e + p e = \Gamma / r $ 54 41 55 51 54 41 55 51 51 51 51 68 64 41 55 51 51 51 61
	$egin{array}{c} P_2 \\ P_2 \end{array}$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$

Relation (32) was experimentally confirmed by the studying of the induced electrogyration, piezogyration and electrooptical effects in the ammonium Rochelle salt crystals which undergo ferroelectric-ferroelastic phase transition at T_c =110K with the change of symmetry 222 \leftrightarrow 2. The value of the piezooptical constant π_{55} is the same as previously obtained in [45]. Calculated from the dependencies presented on Fig.10 values of the piezogyration, electrogyra-

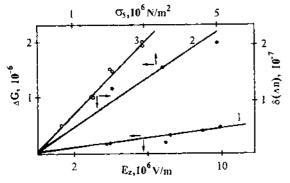


Figure 10. Dependence of the change of gyration tensor on electrical field (1) and mechanical stress (2) and birefringence on electrical field (3) in ammonium Rochalle salt (λ =632.8nm, T=293K).

tion and electrooptical constants are β_{55} =4.6×10⁻¹³m²/N, γ_{52} =5.5×10⁻¹³m/V and r_{52} =2.1×10⁻¹²m/V respectively. Relation for this constants $\beta_{55}/\pi_{55} = \gamma_{52}/r_{52}$ =0.22 is in good agreement with our approach [46].

4. Conclusions

In this article the analysis and experimental studying of the interaction of the circularly polarized waves as the eigen waves of gyrotropic crystals is made. The new effects such as two-beam refraction of the circular waves, acousto-gyration diffraction of light, interferencion of the circularly polarized waves are phenomenologycaly predicted and experimentally studied. The phenomenologycal analysis and experimentally studying of the different types of optical activity that appears as the result of interaction of circular waves at the influence of electrical and magnetic field as well as mechanical strain is presented. The relations for the electro-,

piezooptical and electro-, piezogyration effects in paraphase of the ferroelectrics-ferroelastics are obtained. One can conclude that crystallooptics of the circularly polarized waves could be considered as the separate branch of traditional crystallooptics.

References

- 1. Vlokh R.O., Parkhomenko O.V., Pyatak Yu.A., Skab I.P.Ukr.J. of Phys. (1990) **35** N5 p.680-682 (in Ukrainian).
- 2. Vlokh R., Pyatak Yu., Skab I. Ferroelectrics (1992) **126** p.243-246.
- 3. Born M., Wolf E. Principles of optics. Pergamon Press (1964) 855p.
- 4. Sirotin Yu.A., Shaskolskaya M.P. The basis of the crystallophysics (1975) Moscow "Nauka" 680p. (in Russian).
- 5. Poggendorff J.C. Pogg.Ann. (1839)48 p.461.
- 6. Haidinger W. Wiener Ber. (1854) 16 p.129.
- 7. Voigt W. Phys Z. (1905) 6 p.672.
- 8. Pockels F. Lehrbuch der Kristallooptik. Leipzig Verlag Teubner (1906).
- 9. Bloembergen N., Shih H. Optics communications (1969) 1 N2 p.70-72.
- 10.Khatkevich A.G., Kurilkina S.N. J. Appl. Spec. (1989) N6 p.1005-1009 (in Russian).
- 11.Belsky A.M., Khapalyuk A.P. Optics and spectroscopy (1978) **44** N3 p. 540-544 (in Russian).
- 12.Khatkevich A.G. Optics and spectroscopy (1979) 46 N3 p.505-510 (in Russian).
- 13.Illarionov A.I., Stroganov V.I., Kdyarov B.I. Optics and spectroscopy (1980) **48** N3 p.678-585 (in Russian).
- 14.Bokut B.V., Girgel S.S. J.Appl.Spec. (1982) **36** N4 p.666-669(in Russian).
- 15.Schell A.J., Bloembergen N. J.Soc.Am. (1978) **68** N8 p.1093-1106.
- 16.Schell A.J., Bloembergen N. Phys.Rev.(A) (1978) **A18** N6 p.2592-2602.
- 17. Portigal D.L., Burstein E. J. Opt. Soc. Am. (1972) **62** N7 p. 859-864.

- 18. Vlokh R., Pyatak Yu., Skab I. Ferroelectrics (1992) **126** p.243-246.
- 19. Vlokh R.O., Skab I.P. Ukr.J. of Phys. (1993) **38** N8 p.988-989 (in Ukrainian).
- 20. Vlokh O.G., Zaryk A.V. News of Lviv Univ. (1982) **16** p.13-34 (in Ukrainian).
- 21. Vlokh O.G., Zaryk A.V. Ukr.J. of Phys. (1981) **26** N7 p.1087-1090 (in Russian).
- 22. Vlokh O.G., Golovey M.I., Zaryk A.V. Ukr. J.of Phys.(1982)27 N4p.595-598 (in Russian)
- 23. Vlokh O.G., Kobulyansky V.B., Lazko L.A. Ukr.J. of Phys. (1974) **19** N10 p.1661-1665 (in Ukrainian).
- 24. Vaynshtayn B.K. Modern crystallography. Moscow "Nauka" (1981) 495p. (in Russian).
- 25. Vlokh R.O., Skab I.P., Pyatak Yu.A. Phys. of Solid State (1991) **33** N9 p.2720-2724.
- 26. Vlokh R.O., Pyatak Yu.A., Skab I.P. Ukr.J. of Phys. (1992) **37** N3 p.365-367.
- 27. Vlokh R.O., Pyatak Yu.A., Skab I.P. Optics and spectroscopy (1991) **70** N1 p.243-244 (in Russian).
- 28. Vlokh O.G. Ukr. J. of Phys. (1970) **15** N5 p.759-763 (in Ukrainian).
- 29. Vlokh O.G., Klepach M.I., Kushnir O.S., Shopa Ya.I Ukr. J. of Phys. (1986) **31** N8 p.1228-1231 (in Russian).
- 30. Vlokh R.O., Pyatak Yu.A., Skab I.P. Crystallography (1989) **34** N2 p.498.
- 31. Vinecky V.L., Kukhtarev N.V., Odoulov S.G., Soskin M.S. Usp.Fiz.Nauk (1979) **129** p.113-136 (in Russian).
- 32. Kukhtarev N.V. Anysotropy of the self-diffraction in the photorefractive gyrotropic crystals. Inst. of Phys Uk. Acad. of Sc. (1985) N85/8, Kiev 23p. (in Russian).

- 33. Kukhtarev N.V., Muravev V.V., Semenec T.I. Polarization performances of the self-diffraction in photorefractive crystals. Inst. of Phys Uk.Acad. of Sc. (1986) N19, Kiev 21p. (in Russian).
- 34.Balakshiy V.I., Parigin V.N., Chircov L.E. Physical basis of the acoustooptics. (1985) Moscow "Radio i svyaz" 279p. (in Russian).
- 35.Vlokh R.O., Skab I.P., Pyatak Yu.A.Rep. of Acad. of Sciense of Ukr. SSR (1991) N7 p.39-41 (in Ukrainian).
- 36. Vlokh R.O., Pyatak Yu. A., Skab I.P. News of the Acad. Sciens. of USSR (1989) **53** N7 p.1339-1341 (in Russian).
- 37. Vlokh R., Vlokh O., Pyatak Yu., Skab I. Ferroelectrics (1990) **110** p.1117-1121.
- 38.Vlokh O.G.News of Lviv Univ. (1982) **16** p.5-12 (in Ukrainian).
- 39. Zheludev I.S., Vlokh O.G., Sergatyook V.A. Ferroelectrics (1985) **63** p.97-105.
- 40. Vlokh O.G., Sergatyook V.A. Rep. Of Acad. Scins. of USSR (1986) **291** p.832-835 (in Russian).
- 41. Vlokh O.G., Sergatyook V.A. Ferroelectrics (1988) **80** p.313-316.
- 42. Vlokh O.G., Vlokh R.O. Optics and spectroscopy (1989) **69** N1 p.225-227 (in Russian).
- 43. Vlokh O.G., Vlokh R.O. Optics and spectroscopy (1990) **69** N2 p.458-459 (in Russian).
- 44. Vlokh R.O. Ukr. J. of Phys. (1989) **34** N12 p.1809-1811 (in Ukrainian).
- 45.Ivanov N.R., Khusravov D.,Shuvalov L.A., Schagina N.M. News of Acad.Scien. of USSR(1979)43 N8 p.1691-1701(in Russian).
- 46.Berezhnoy I.V., Vlokh R.O. Phys. of Solid State (1988) **30** N7 p.2223-2225 (in Russian)